Reconstruction 3D à partir de coupes 2D

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Projet Geometrica, INRIA Sophia Antipolis

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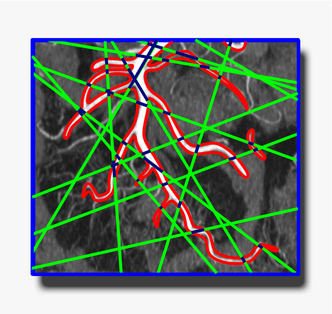
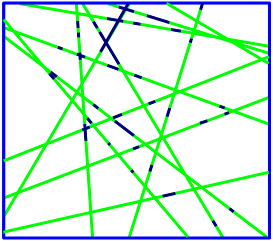
Reconstruction Problem





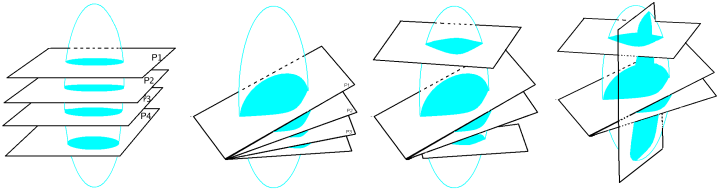
Motivations

(a) Input: 2D Images + Orientation of the captor

Motivations

(b) Actual Technology

Different cross-sections positions



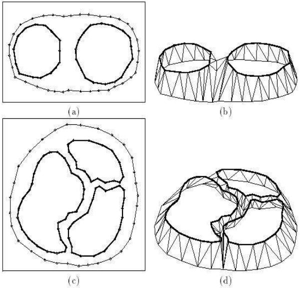
Parallel planes

Non-parallel serial sequence of planes

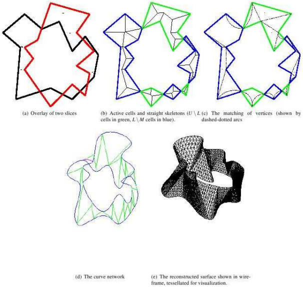
Arbitrary cutting planes

Geometric Methods (parallel case)

Most of methods use the superposition of the contours.



Bajaj et al. [1996]



Barequet et al. [1996-2004]

Outline

Planar Cross-Sections of an Object

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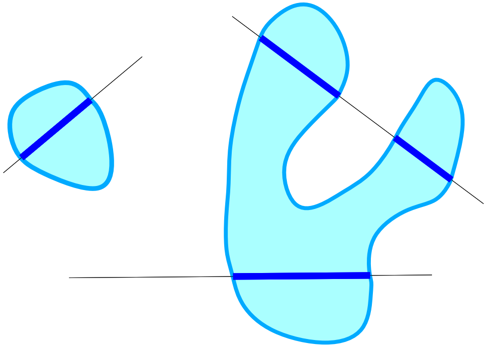
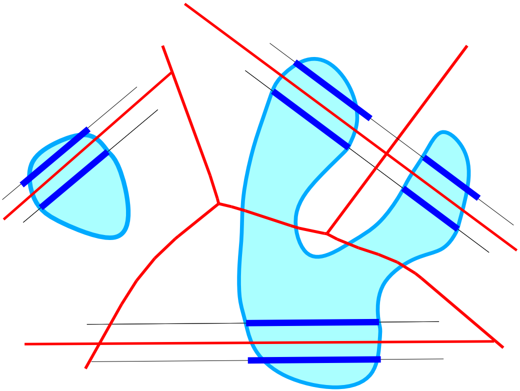
A 3-manifold O is cut by a set of cutting planes P.



Cross-sections S are composed of contractible sets (called sections).

Voronoi Diagram of Duplicated Sections

The Voronoi diagram of **duplicated** sections, $Vor(S)$, forms a covering of O.



Nerve of the restriction of $Vor(S)$ to O

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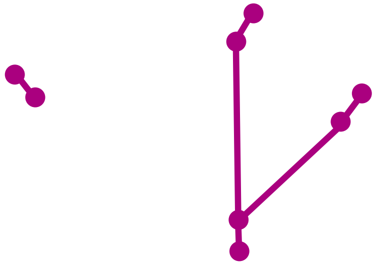
Definition of $Nerve(S)$: To each duplicated section we associate a vertex. Two vertices are linked in $Nerve(S)$ if the commun face of their Voronoi cells intersects O.



$Nerve(S)$ may contain 0, 1, 2 or 3 dimensional simplices.

Topological Condition and Nerve Theorem

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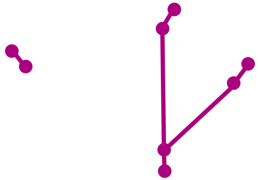
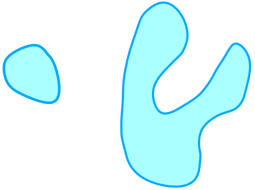
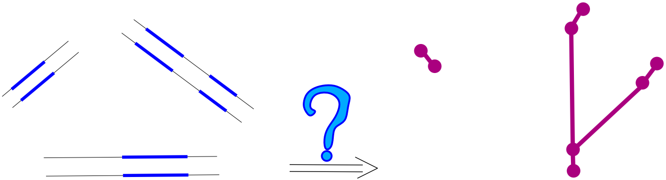
Nerve Theorem: If P is sufficiently dense so that
$\forall$ face f of $Vor(S)$: $f \cap O$ is **contractible** (Condition T)
then $Nerve(S)$ has the same homotopy type as O.





Reconstruction Problem

We now want to approximate O **knowing only the sections**.



Knowing the section, is it possible to find a reconstruction topologically equivalent to $Nerve(S)$ and so to O?

Definitions

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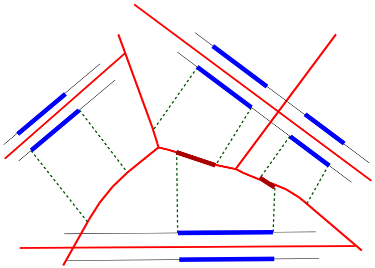
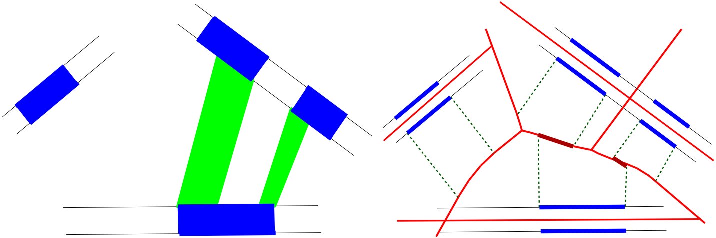
Duality: between Delaunay balls and Delaunay simplices
tangent balls with centers on the **lift** of sections



Core of the medial axis

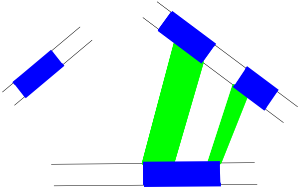
Kernel's definition

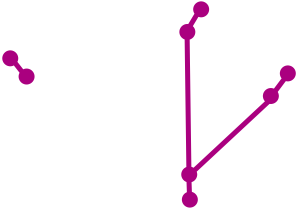


Kernel is the dual of the core of the medial axis

Main Idea

We claim that if P is sufficiently dense then:
Kernel is homotopy equivalent to *Nerve*(S).





Sampling Conditions

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In $Vor(A)$, $shift(A)$ is the "*horizontal*" distance between ∂A and ∂O. $Shift(S) = min_{A \in S} shift(A)$

Condition 1 $Shift(S)$ is sufficiently small.

Condition 2 $\forall$ i-Voronoi face F, if F intersects the lift of all of its sections then $core(F)$ is an i-disk.

these conditions are verified for a sufficiently dense sample of cutting planes.

Correspondence Lemma

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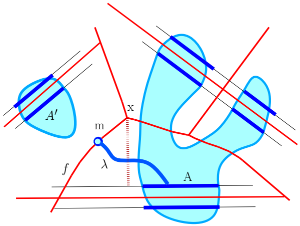
Lemma (Correspondence lemma)

If Conditions T, 1 and 2 are verified, for any Voronoi face F we have:

$$F \cap O \neq \emptyset \iff core(F) \neq \emptyset.$$



Contraction

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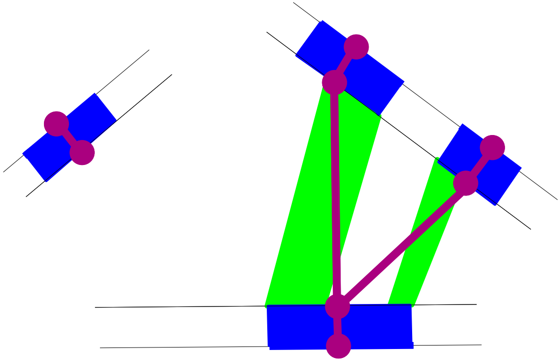
Corollary: According to the correspondence lemma:
the correspondence between the sections is the same in the kernel and the nerve.

Lemma: $\forall$ Voronoi face F, $F^* \cap$ Kernel is either empty or contractible.

Because $F^* \cap$ Kernel is the dual of $core(F)$ which is contractible.

Contraction

Contracting any section to one of its points proves the homotopy equivalence between:
Kernel and *Nerve*(*S*).

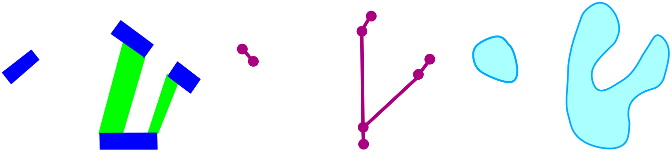


Homotopy theorem

If P is sufficiently dense then the conditions are verified and:

- Kernel and
- $Nerve(S)$ and
- O

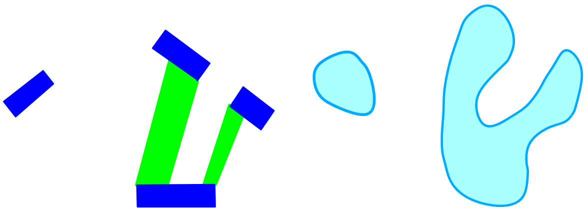
are homotopy equivalent.



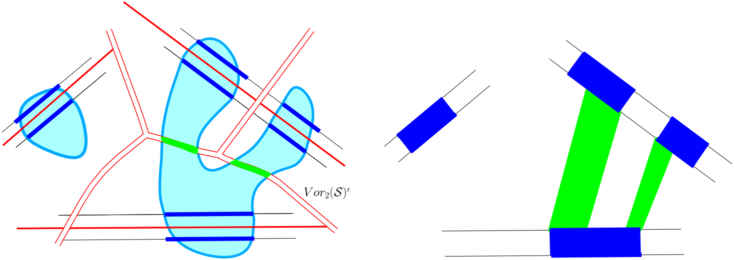
Outline

Homeomorphism

Theorem: Kernel is homeomorphic to O



Sketch of proof



$O \cap Vor(S)^\epsilon$ is homeomorphic to the green part of the kernel.

Conclusion

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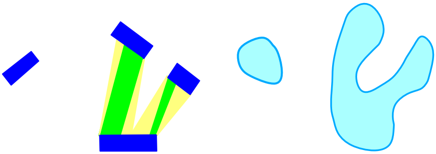
- Generalization of the classical overlapping criterion
- Topologically correct solution for the correspondence problem
- Justification of most of existing methods in parallel case
- Justification of our generalized Delaunay-based reconstruction (SGP 07-08)



Thank you