Journées de Géométrie Algorithmique, January 2009

Analysis of Scalar Fields over Point Cloud Data

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Géometrica Group
INRIA Saclay

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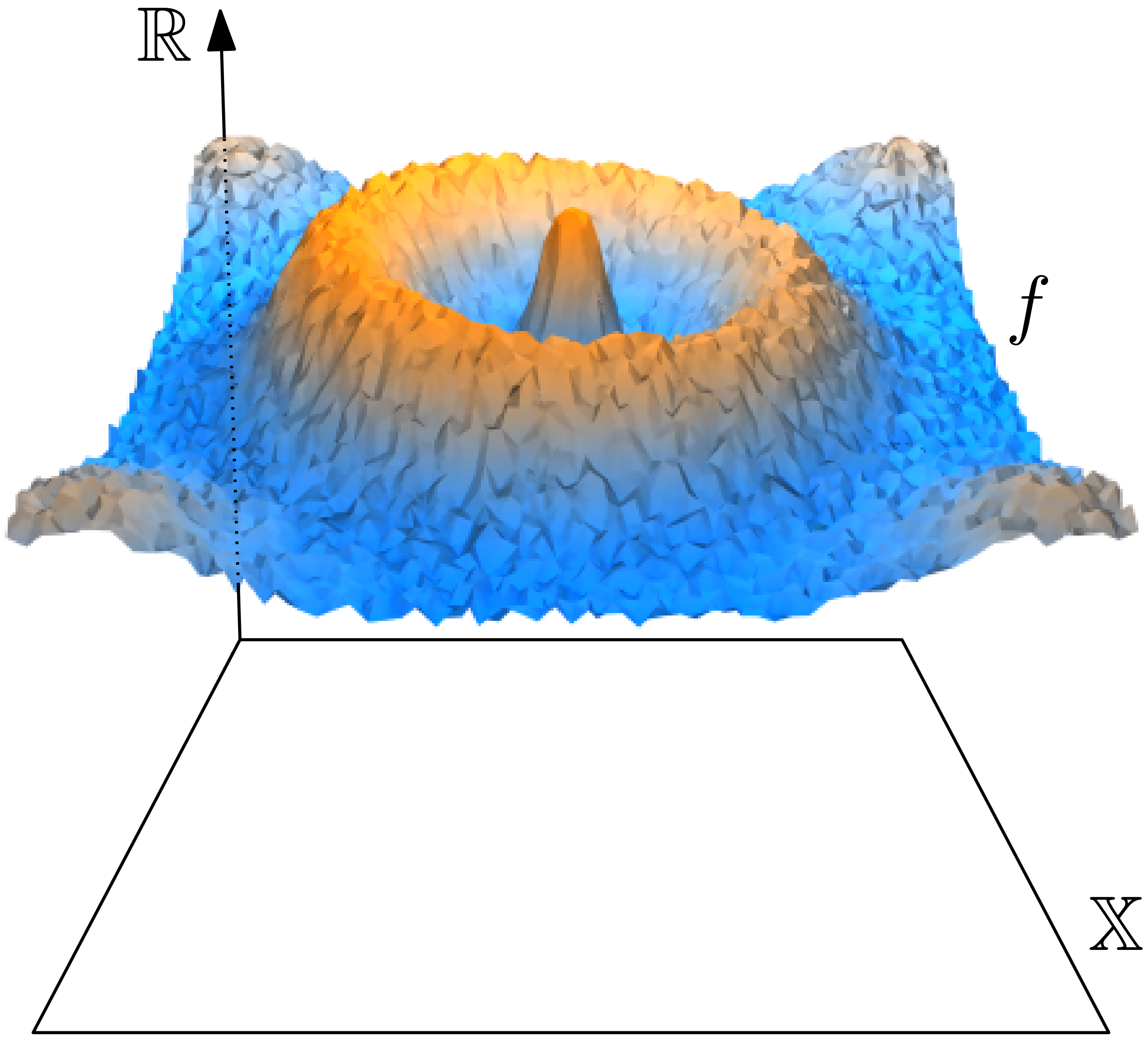
Scalar Field Analysis

Setting: $\mathbb{X}$ topological space, $f : \mathbb{X} \to \mathbb{R}$



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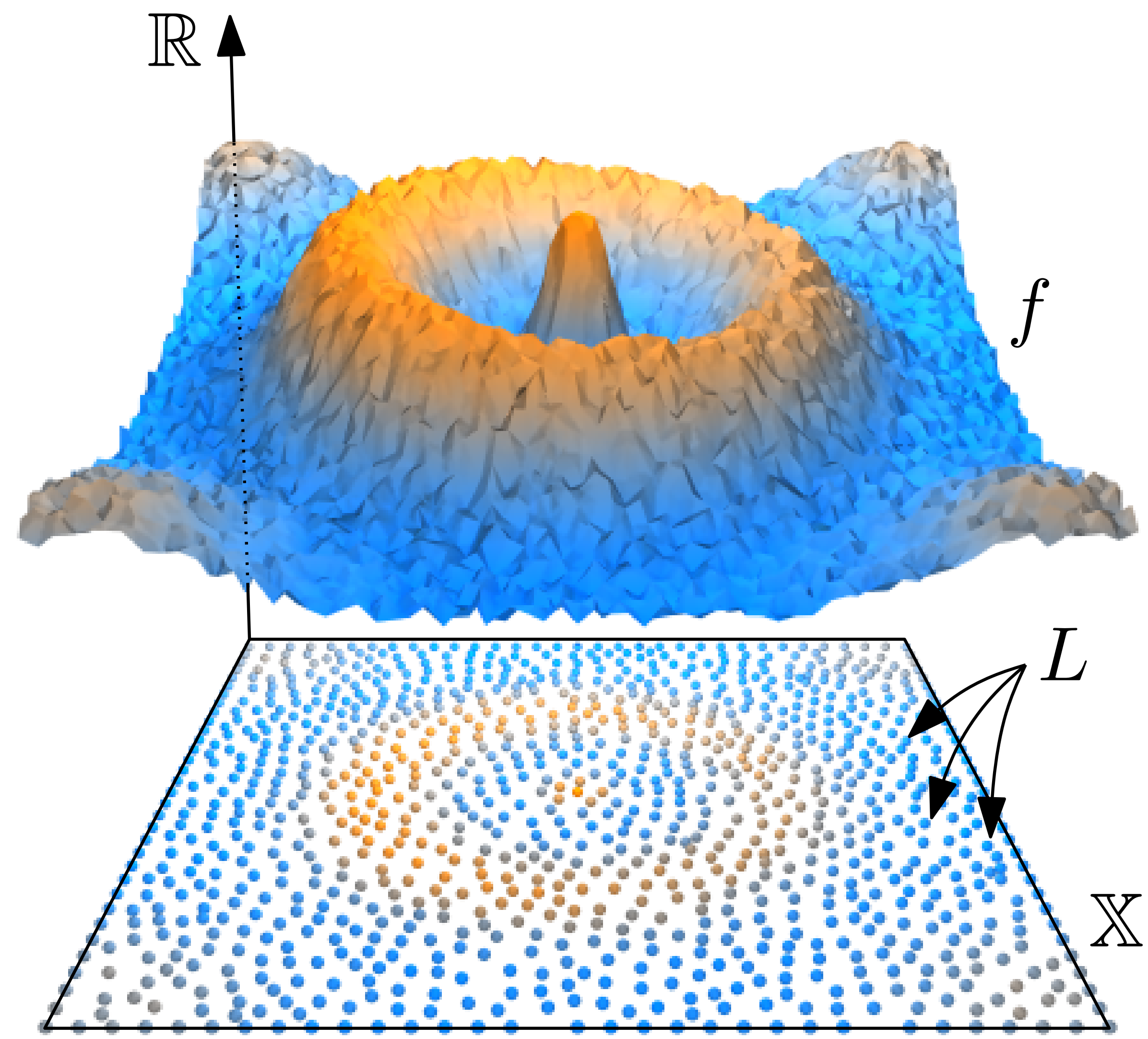
Input: A finite sampling L of $\mathbb{X}$, the values of f at the sample points

Goal: Analyze landscape of graph(f):

- *prominent* peaks/valleys



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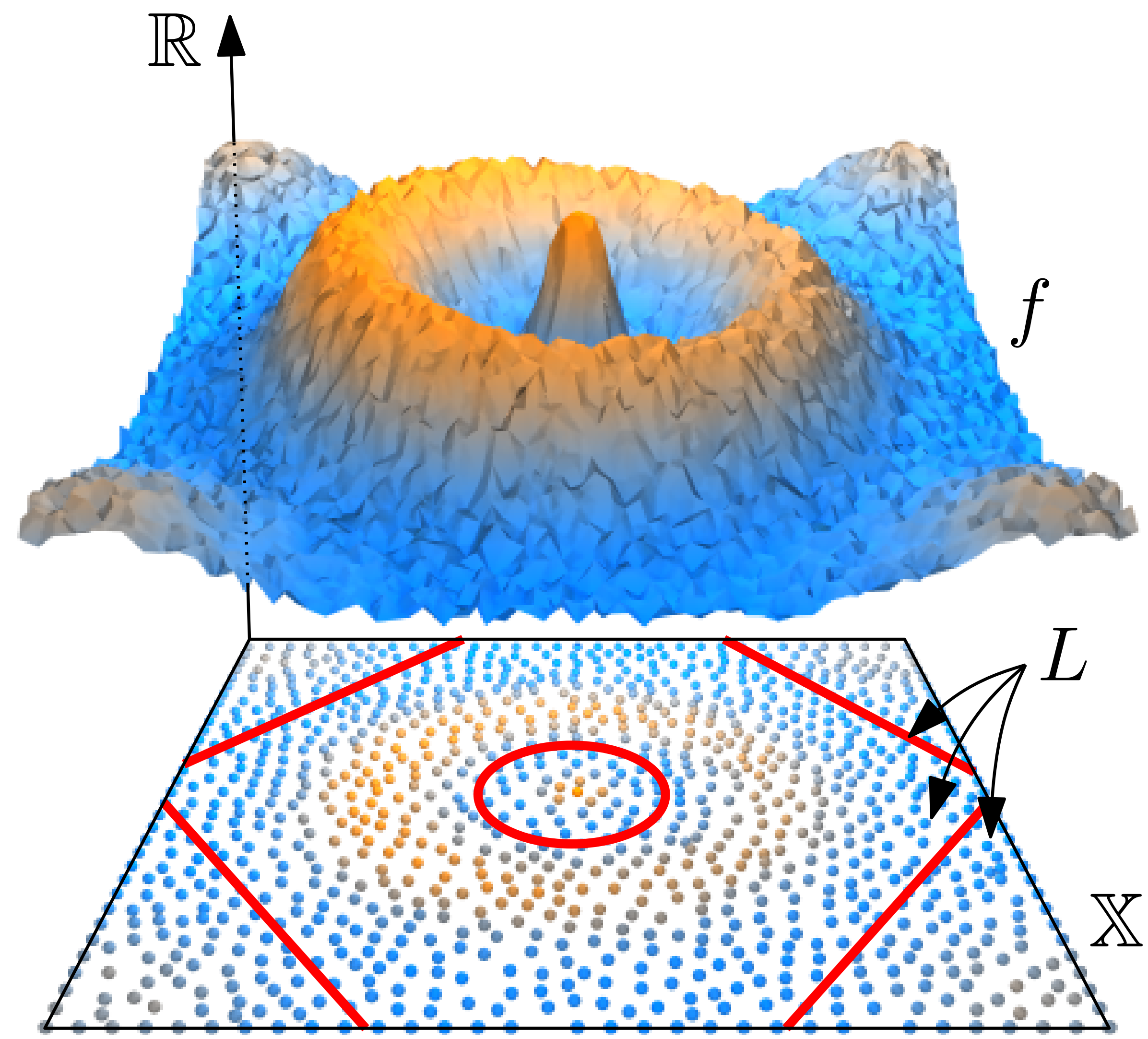


Motivating Applications

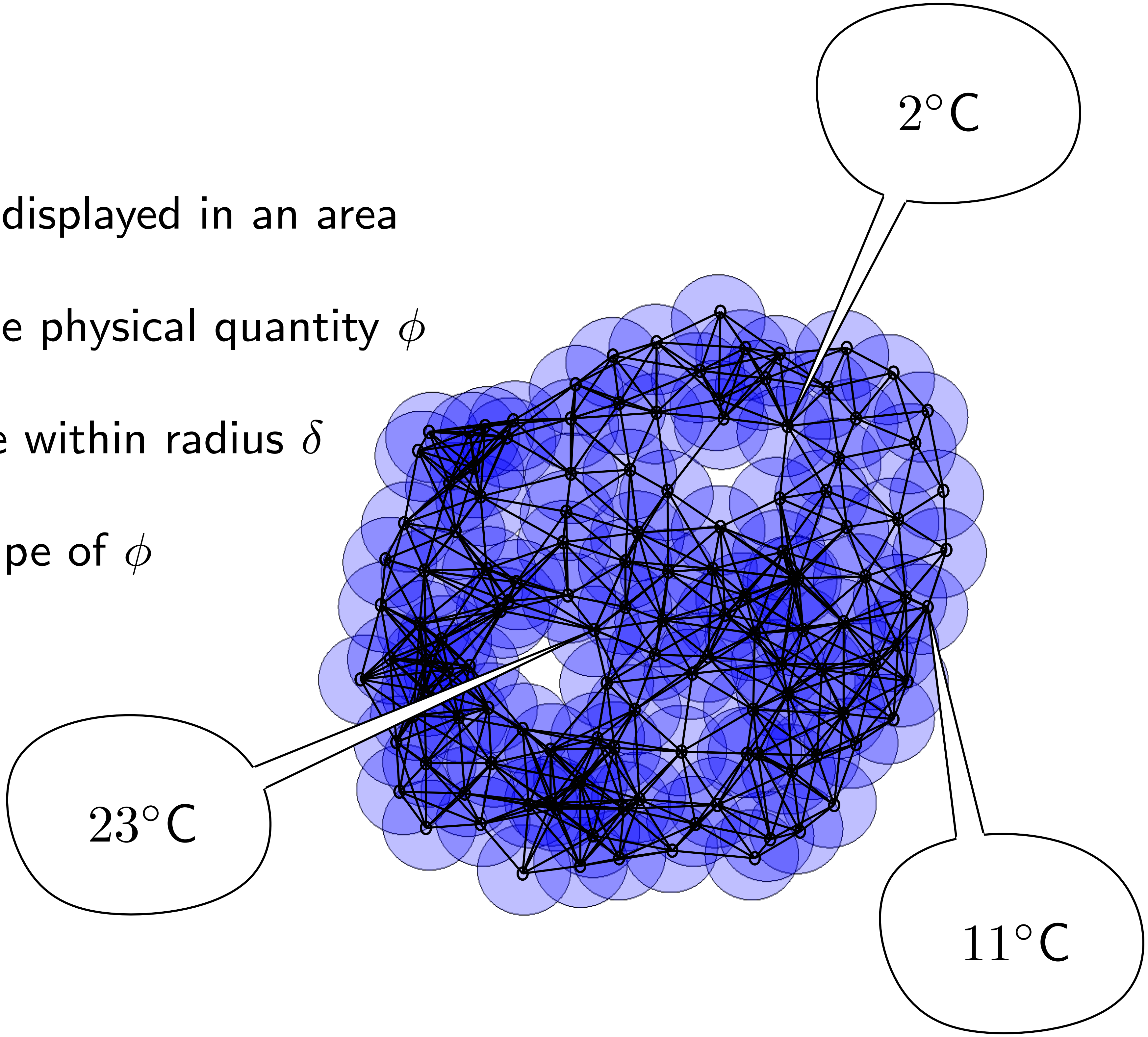
- Sensor networks:

  - collection of sensors displayed in an area

  - sensors measure same physical quantity ϕ

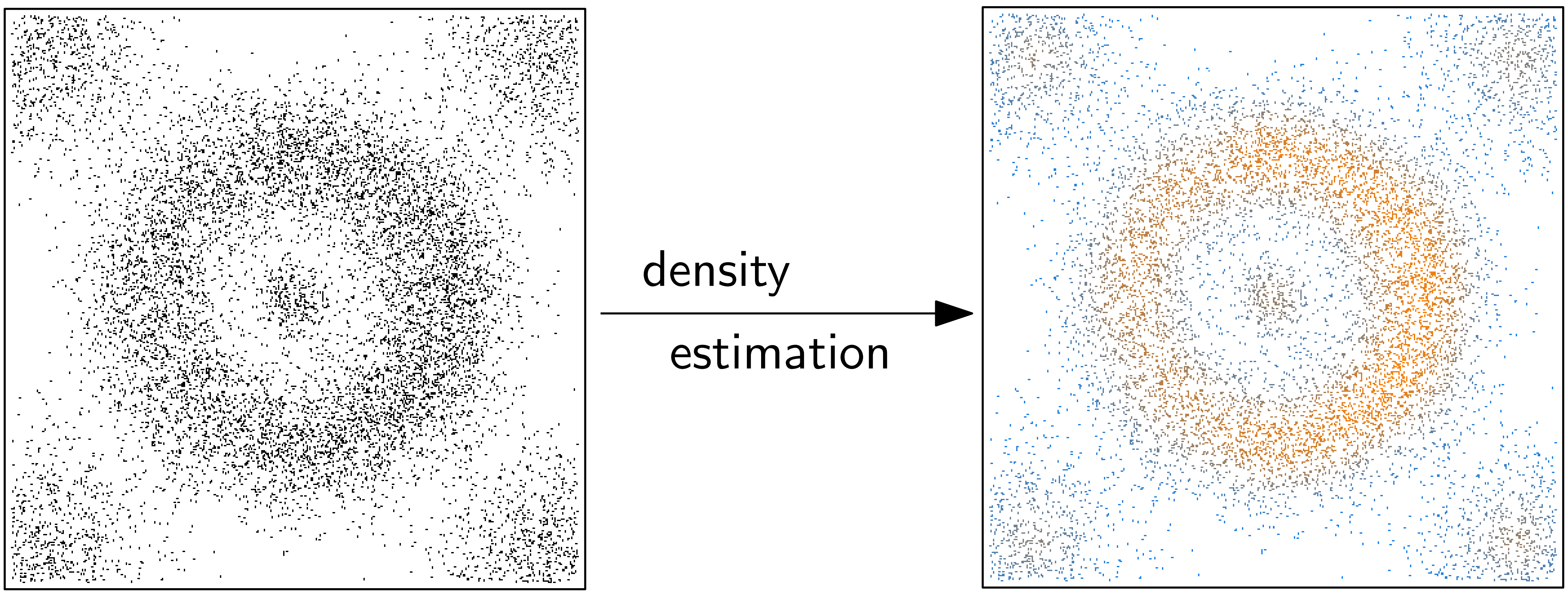
  - sensors communicate within radius δ

  Goal: Analyze landscape of ϕ



Motivating Applications

- Sensor networks:
- Unsupervised learning:
  - data points drawn at random from some unknown density distribution f
  - approximate f through some density estimator $\hat{f}$
  - cluster data points according to *prominent* basins of attraction of $\hat{f}$

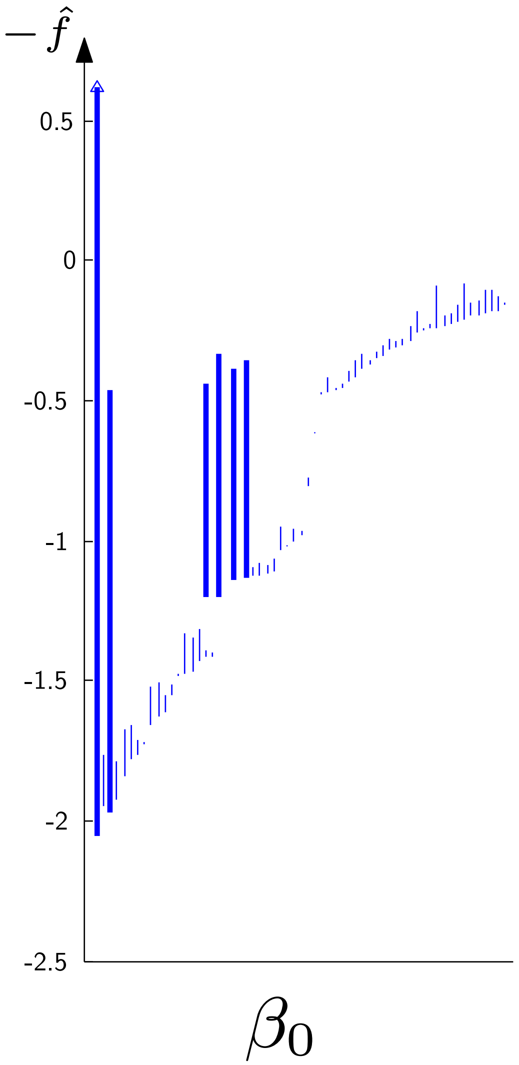


Persistence-Based Approach

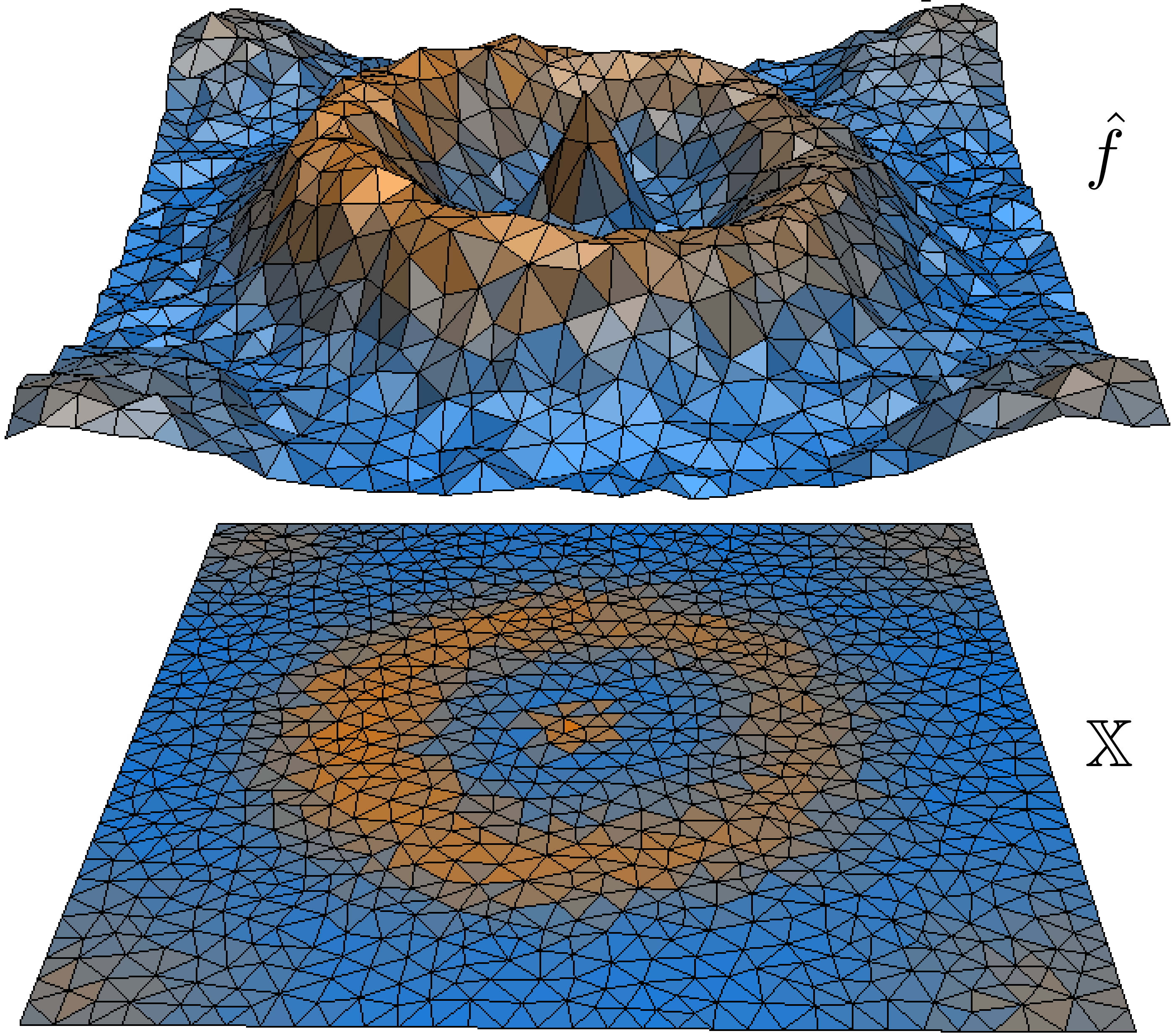
Assumptions: $\mathbb{X}$ triangulated space, $f : \mathbb{X} \to \mathbb{R}$ Lipschitz continuous

$\to$ build PL approximation $\hat{f}$ of f

$\to$ apply persistence algo. to $\pm\hat{f}$ [Edelsbrunner, Letscher, Zomorodian '00]



(6 prominent peaks)



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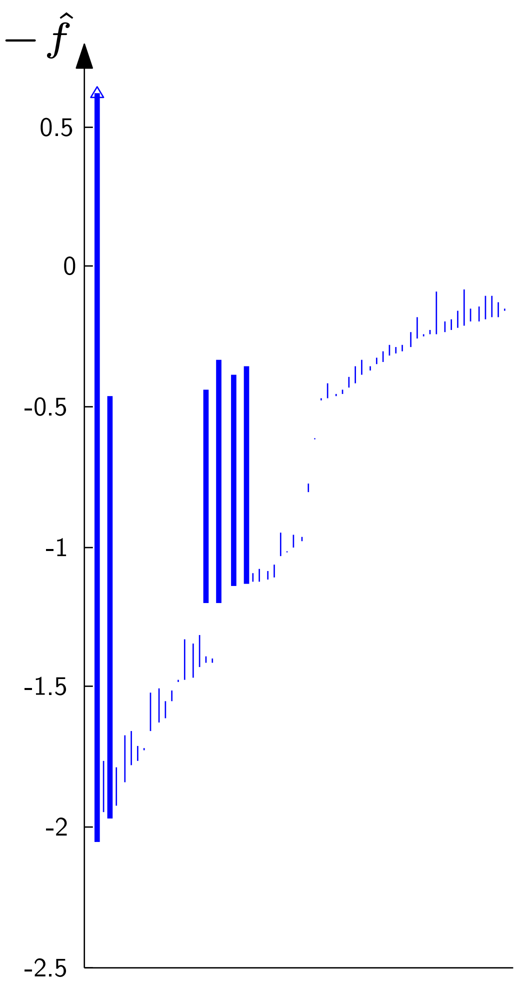
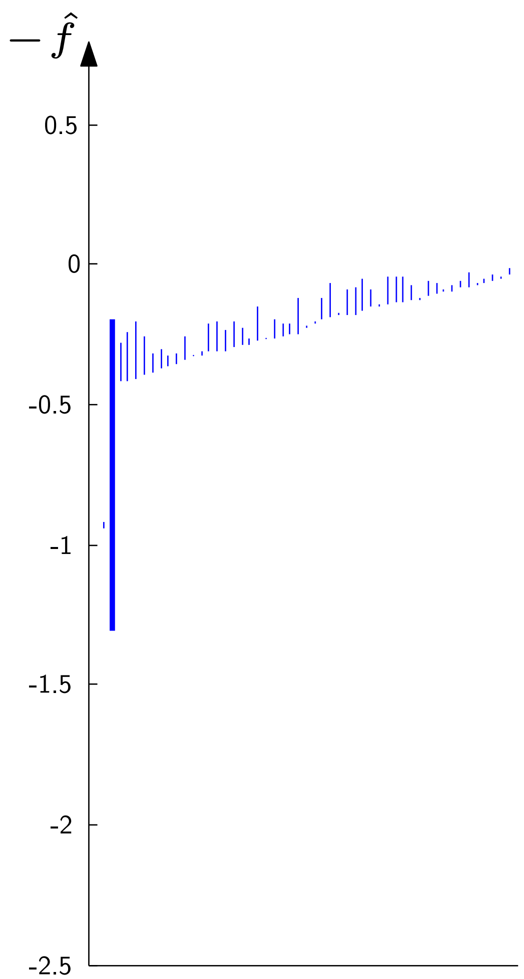


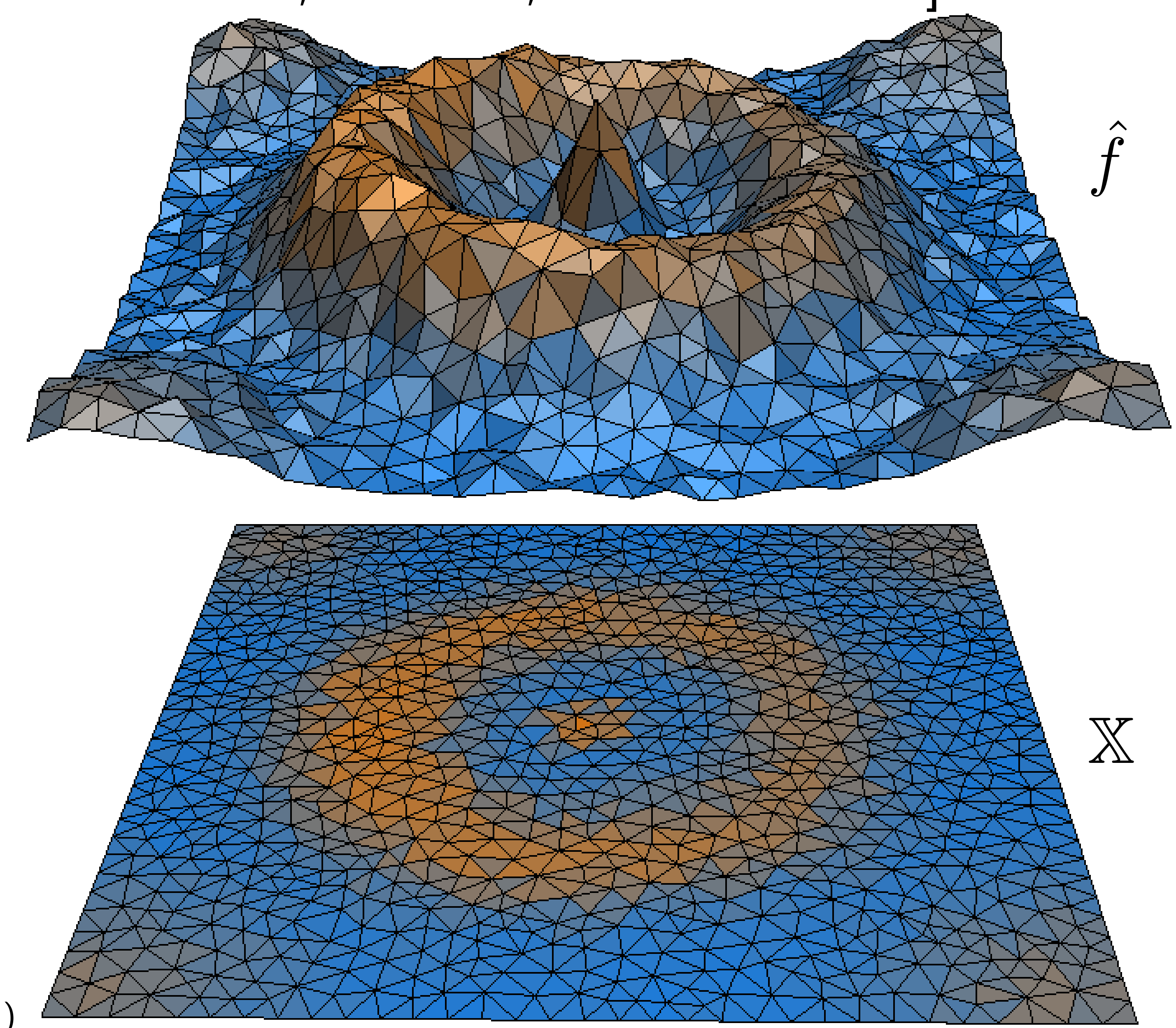
β_0

(6 prominent peaks)



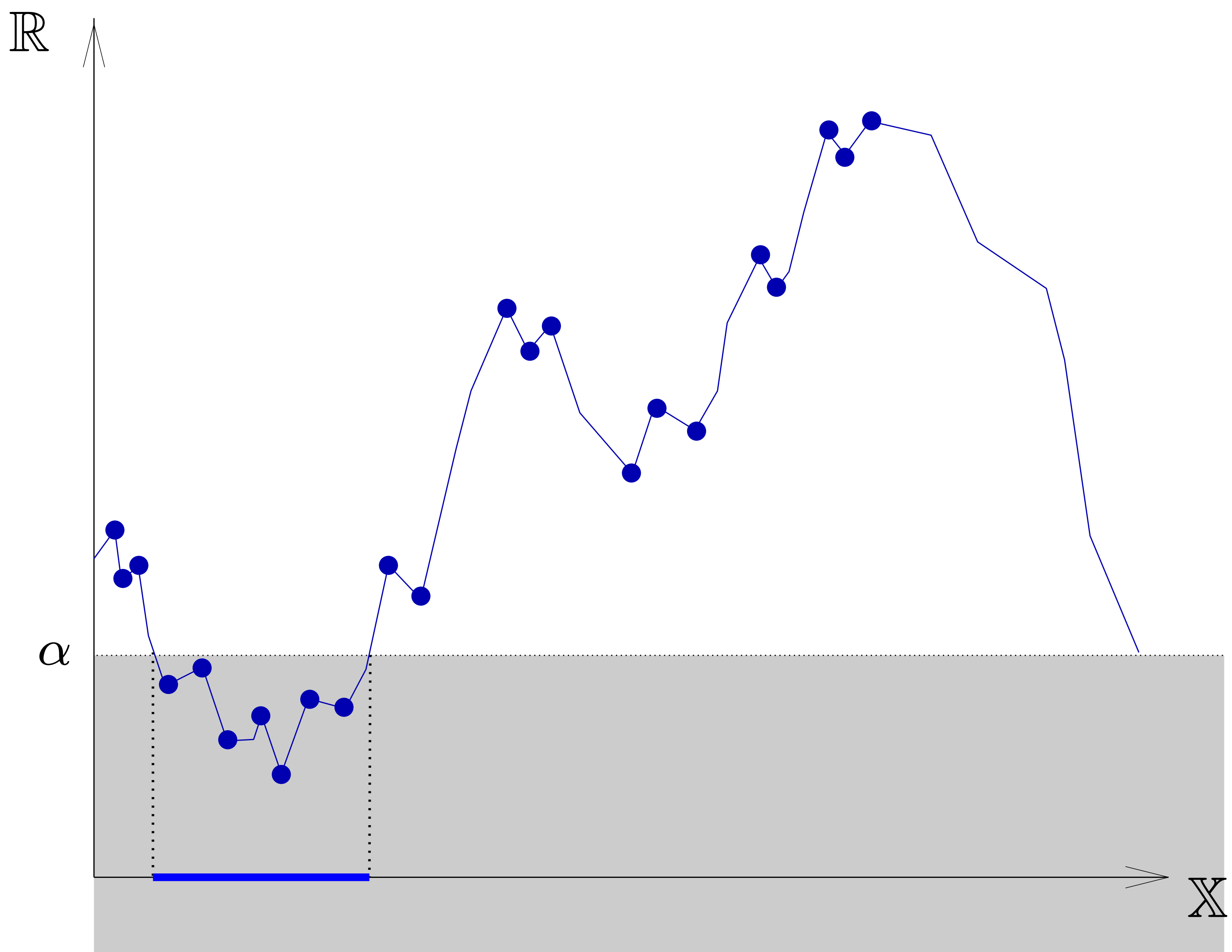
β_1

(ring-shaped basin of attraction)



Persistence-Based Approach in a nutshell...

- evolution of topology of sub-level sets $\hat{f}^{-1}((-\infty, \alpha])$ as α spans $\mathbb{R}$.



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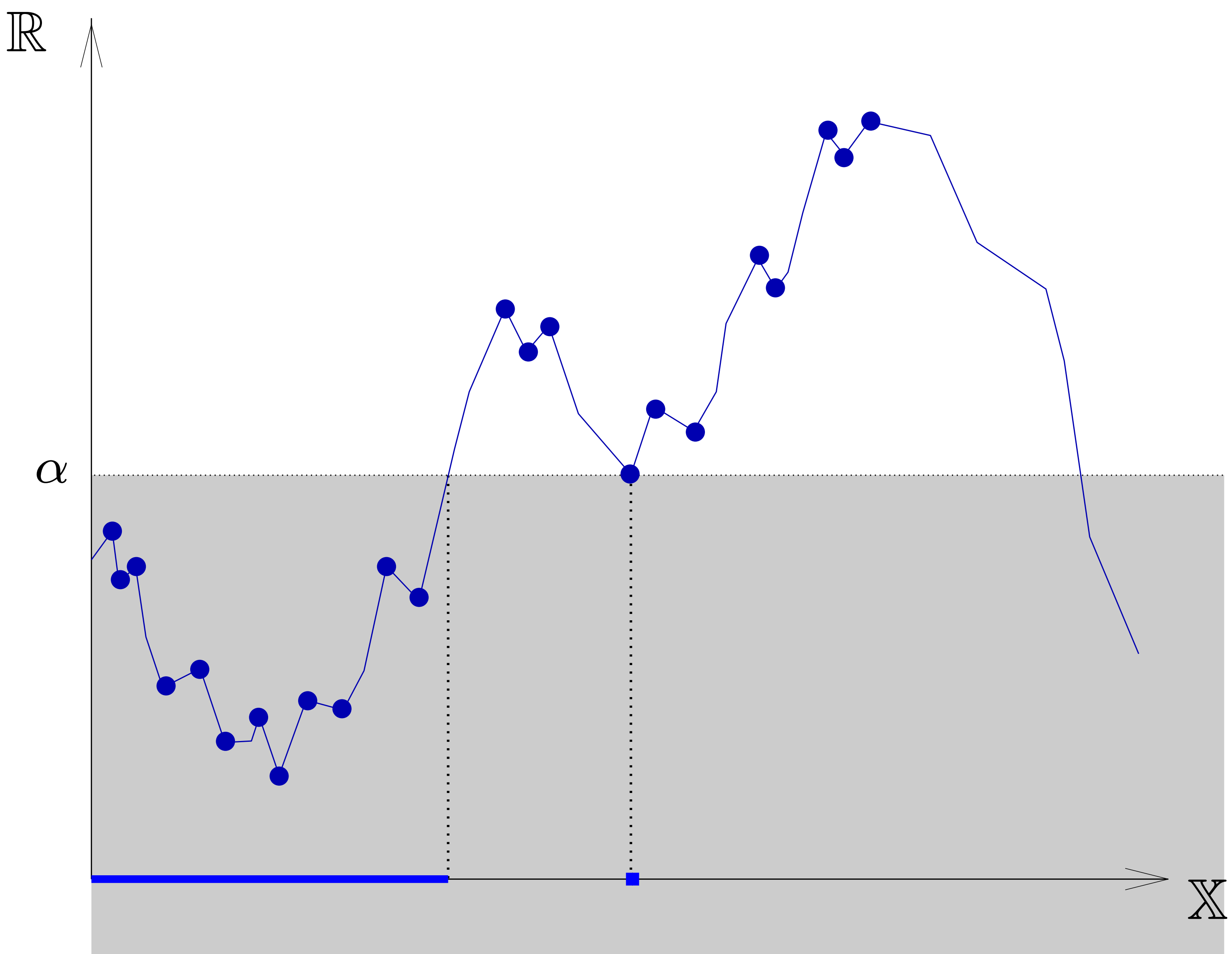

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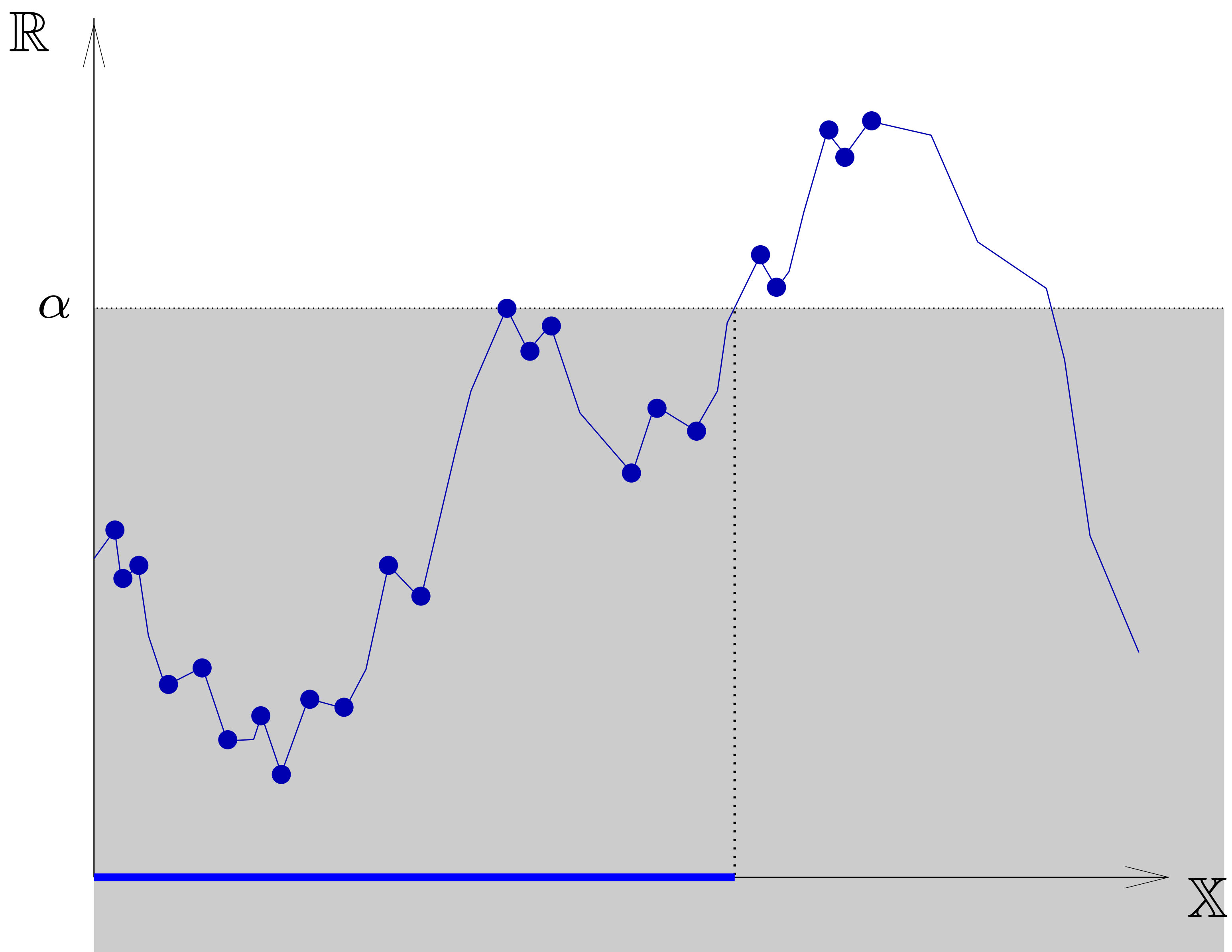
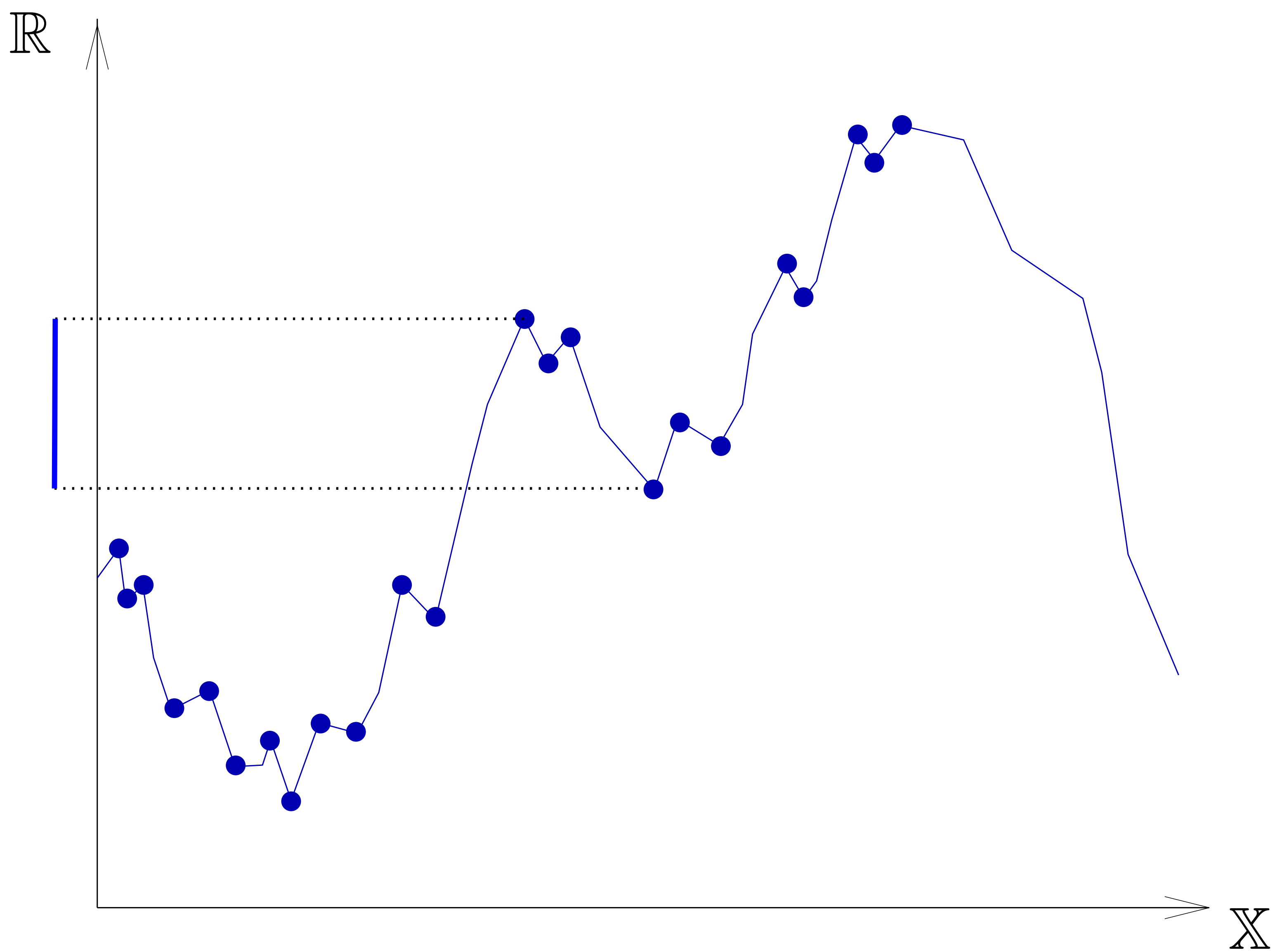


Persistence-Based Approach in a nutshell...

- evolution of topology of sub-level sets $\hat{f}^{-1}((-\infty, \alpha])$ as α spans $\mathbb{R}$.

- finite set of intervals (barcode) encode birth/death of homological features.



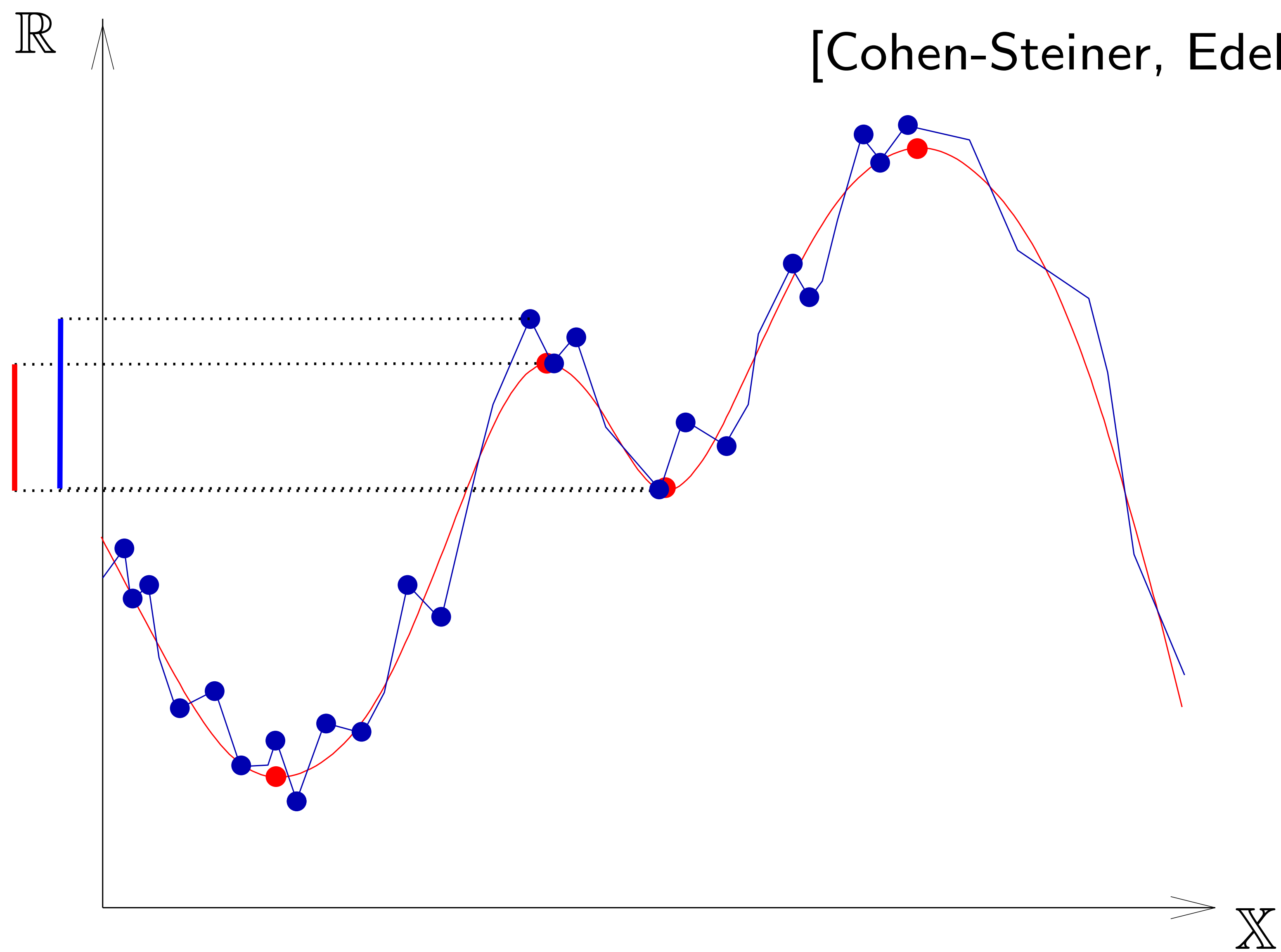
Persistence-Based Approach
in a nutshell...

- evolution of topology of sub-level sets $\hat{f}^{-1}((-\infty, \alpha])$ as α spans $\mathbb{R}$.

- finite set of intervals (barcode) encode birth/death of homological features.

- barcode of $\hat{f}$ is close to barcode of f provided that $\|\hat{f} - f\|_\infty$ is small.

[Cohen-Steiner, Edelsbrunner, Harer '05]



$\longrightarrow$ what if no triangulation of $\mathbb{X}$ is available?

- no geographic information,
- high-dimensional spaces,
- intrinsically curved spaces,
- non-triangulable spaces.

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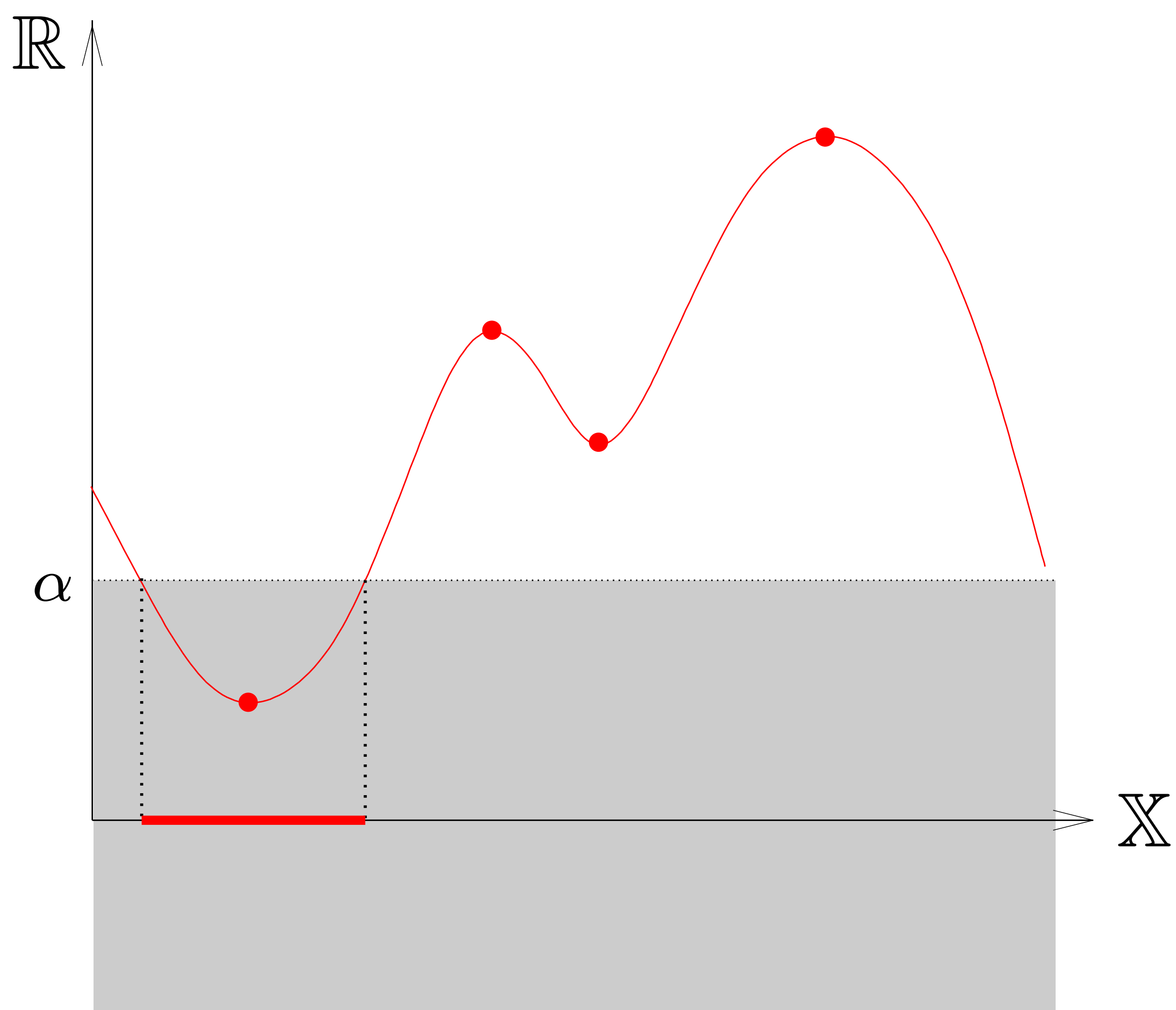

$\longrightarrow$ Approximate sub-level sets of f by other means

Approximation of Sub-Level Sets

- ~~Triangulation~~

Approximation of Sub-Level Sets

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- Stability of persistence diagrams
  - interleaved spaces have close persistence diagrams

$$
\begin{tikzcd}
\cdots \arrow[r] & \phi_{a+2n\varepsilon} \arrow[r] \arrow[dr] & \phi_{a+(2n+1)\varepsilon} \arrow[r] & \phi_{a+(2n+2)\varepsilon} \arrow[r] \arrow[dr] & \cdots \\
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- **Goal:** Relate level sets to something computable

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- **Goal:** Relate level sets to something computable

First step ....

Unions of Geodesic Balls

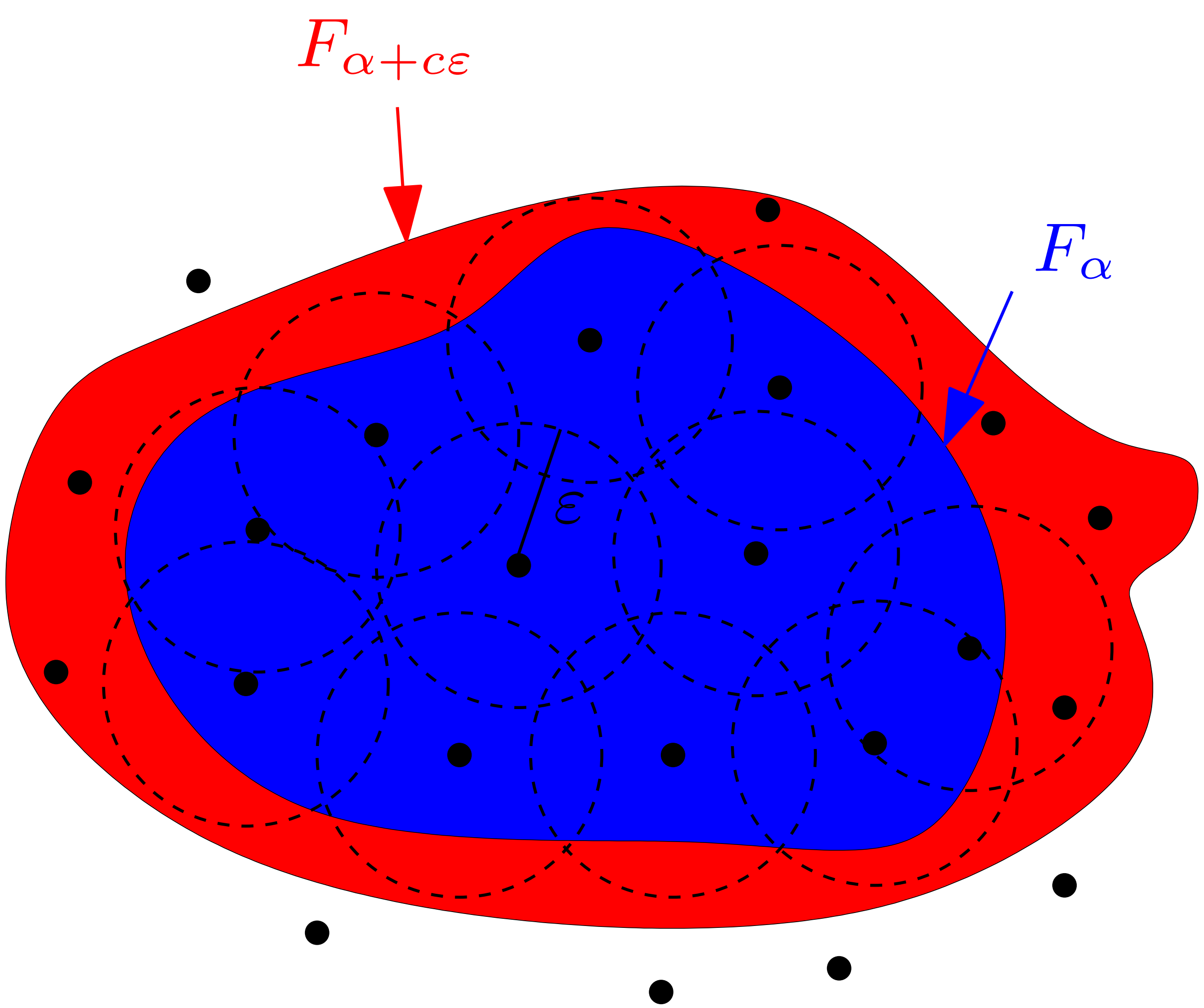
Assumptions: $\mathbb{X}$ Riemannian manifold, $f : \mathbb{X} \to \mathbb{R}$ c-Lipschitz,
L geodesic ε-cover of $\mathbb{X}$, for some unknown $\varepsilon > 0$.

$$F_\alpha := f^{-1}((-\infty, \alpha])$$

$$L_\alpha := L \cap F_\alpha$$

$$L_\alpha^\varepsilon := \bigcup_{p \in L_\alpha} B_{\mathbb{X}}(p, \varepsilon)$$

$$\boxed{\forall \alpha \in \mathbb{R},\ F_\alpha \subseteq L_{\alpha+c\varepsilon}^\varepsilon \subseteq F_{\alpha+2c\varepsilon}}$$



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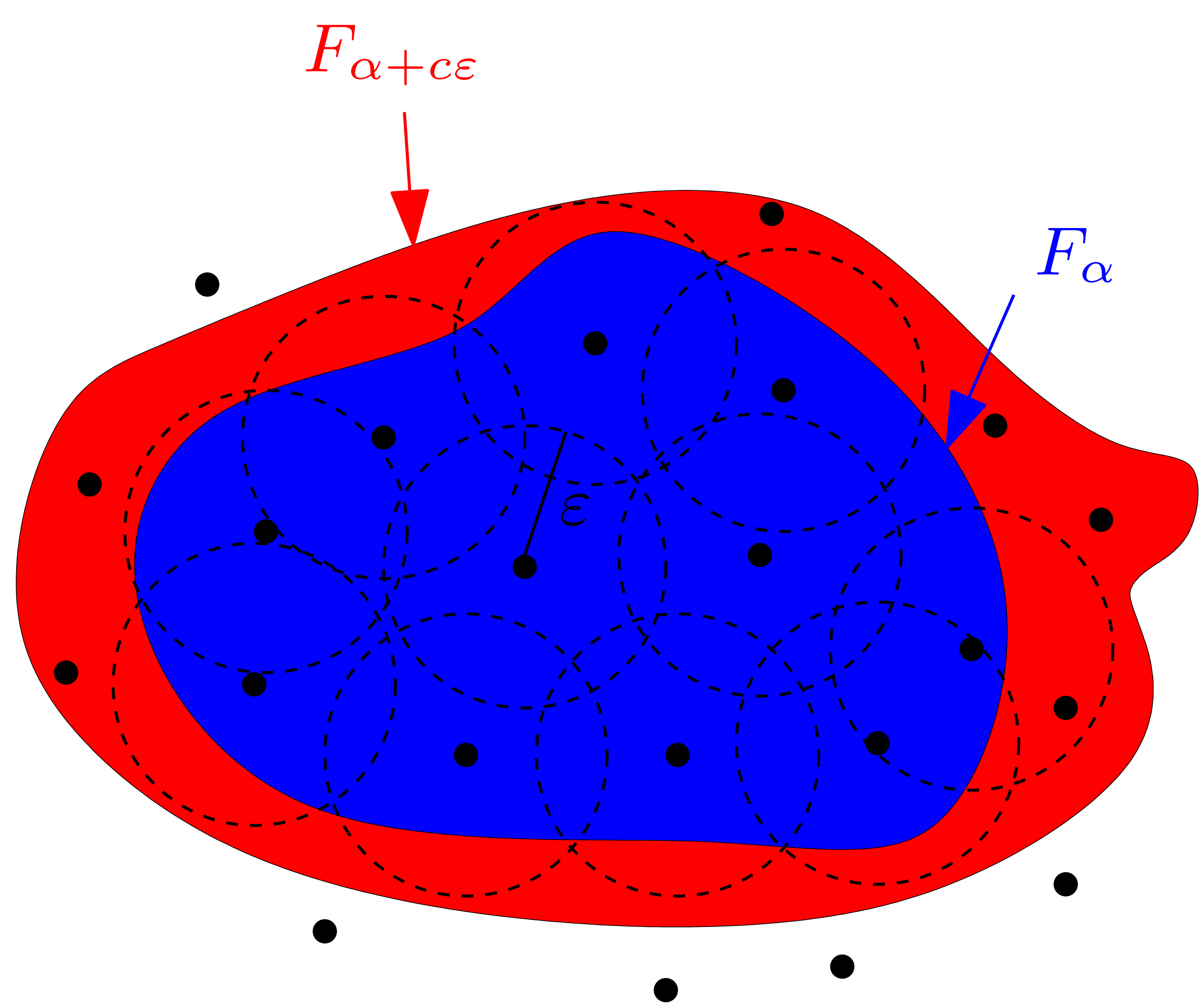
the nested families of spaces
$\{F_\alpha\}_{\alpha\in\mathbb{R}}$ and $\{L^\varepsilon_\alpha\}_{\alpha\in\mathbb{R}}$
are $c\varepsilon$-interleaved

$$\Downarrow$$

their barcodes are $c\varepsilon$-close.



[Chazal, Cohen-Steiner, Glisse, Guibas, Oudot '09]

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Algorithm: (parameter $\delta \geq 0$)

1. Sort the data points such that $f(p_1) \leq f(p_2) \leq \cdots \leq f(p_n)$,
2. For $i = 1, \cdots, n$, build the union of balls $\{p_1, \cdots, p_i\}^\delta$,
3. Apply the persistence algorithm to the nested family of spaces.

$$\{p_1\}^\delta \hookrightarrow \{p_1, p_2\}^\delta \hookrightarrow \{p_1, p_2, p_3\}^\delta \hookrightarrow \cdots \hookrightarrow \{p_1, p_2, \cdots, p_n\}^\delta$$

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$\to$ if $\delta \geq \varepsilon$, get a $c\delta$-approximation of the barcode of f

Unions of Geodesic Balls

Assumptions: $\mathbb{X}$ Riemannian manifold, $f : \mathbb{X} \to \mathbb{R}$ c-[illegible]z,
L geodesic ε-cover of $\mathbb{X}$, for some [illegible] $\varepsilon > 0$.

Algorithm: (parameter $\delta \geq 0$)

1. Sort the data points such that $f($[illegible]$) \leq \cdots \leq f(p_n)$,
2. For $i = 1, \cdots, n$, build the ur[illegible]s $\{p_1, \cdots, p_i\}^\delta$,

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3. Apply the persistence [illegible] to the nested family of spaces.

$$\mathcal{N}^\delta(p_1) \hookrightarrow \mathcal{N}^\delta(p_1 \text{[illegible]} (p_1, p_2, p_3) \hookrightarrow \cdots \hookrightarrow \mathcal{N}^\delta(p_1, p_2, \cdots, p_n)$$

Intractable in practice

Pairs of Rips Complexes

Assumptions: $\mathbb{X}$ Riemannian manifold, $f : \mathbb{X} \to \mathbb{R}$ c-Lipschitz,
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$\to$ idea inspired from [Chazal, Oudot '08]:

$\forall i = 1, \cdots, n$, replace $\{p_1, \cdots, p_i\}^\delta$ by $\mathcal{R}^\delta(p_1, \cdots, p_i) \subseteq \mathcal{R}^{2\delta}(p_1, \cdots, p_i)$

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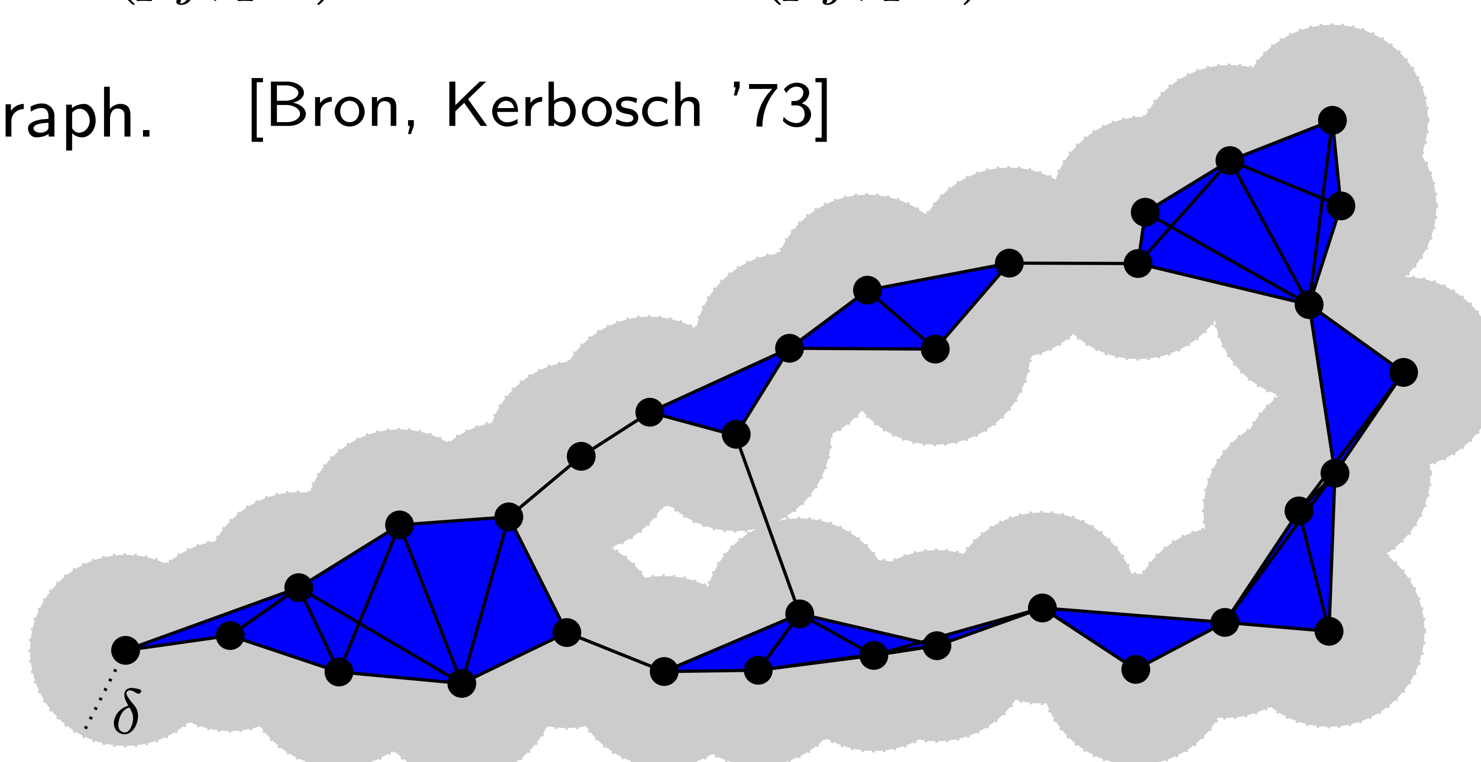
Construction of $\mathcal{R}^{2\delta}(p_1, \cdots, p_i)$:

- connect every pair of points (p_j, p_k) such that $\mathrm{d}_{\mathbb{X}}(p_j, p_k) \leq 2\delta$,

- report all cliques in the graph. [Bron, Kerbosch '73]



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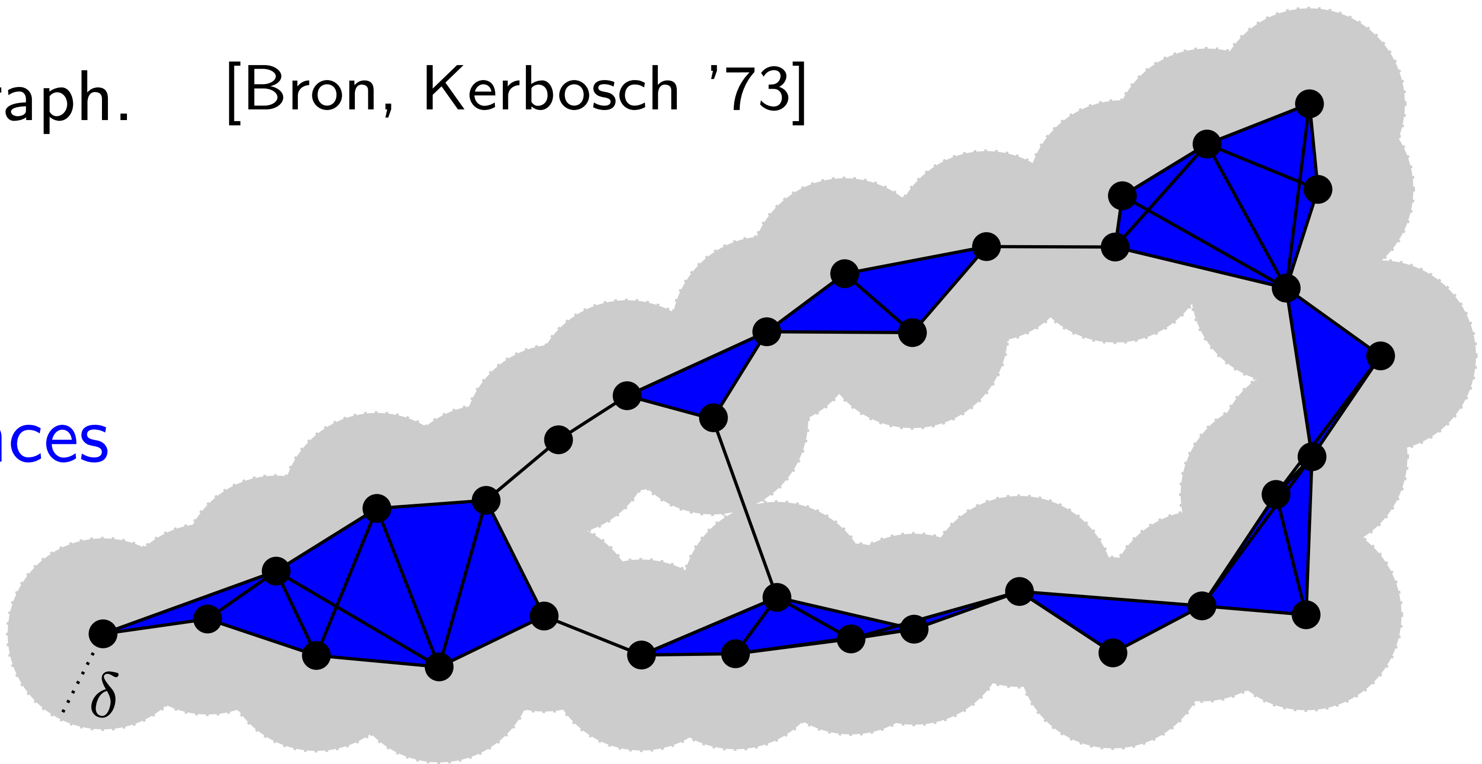
$\rightarrow$ takes $O(n^2 + Cn)$ time

$\rightarrow$ uses only geodesic distances between the data points



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Guarantees:

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\end{tikzcd}$$

$$F_{\alpha_0} \subseteq \{p_1\}^{\delta/2} \subseteq \{p_1\}^{2\delta} \subseteq F_{\alpha_1} \subseteq \cdots\cdots \subseteq F_{\alpha_{n-1}} \subseteq \{p_1, \cdots, p_n\}^{\delta/2} \subseteq \{p_1, \cdots, p_n\}^{2\delta}$$

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$\forall \delta \geq \varepsilon$, $\{F_\alpha\}_{\alpha \in \mathbb{R}}$ and $\{\mathcal{R}^\delta(L_\alpha) \hookrightarrow \mathcal{R}^{2\delta}(L_\alpha)\}_{\alpha \in \mathbb{R}}$ are $2c\delta$-interleaved

$\Downarrow$ [Chazal, Cohen-Steiner, Glisse, Guibas, Oudot '09]

their barcodes are $2c\delta$-close.

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[Cohen-Steiner, Edelsbrunner, Harer, Morozov '09]

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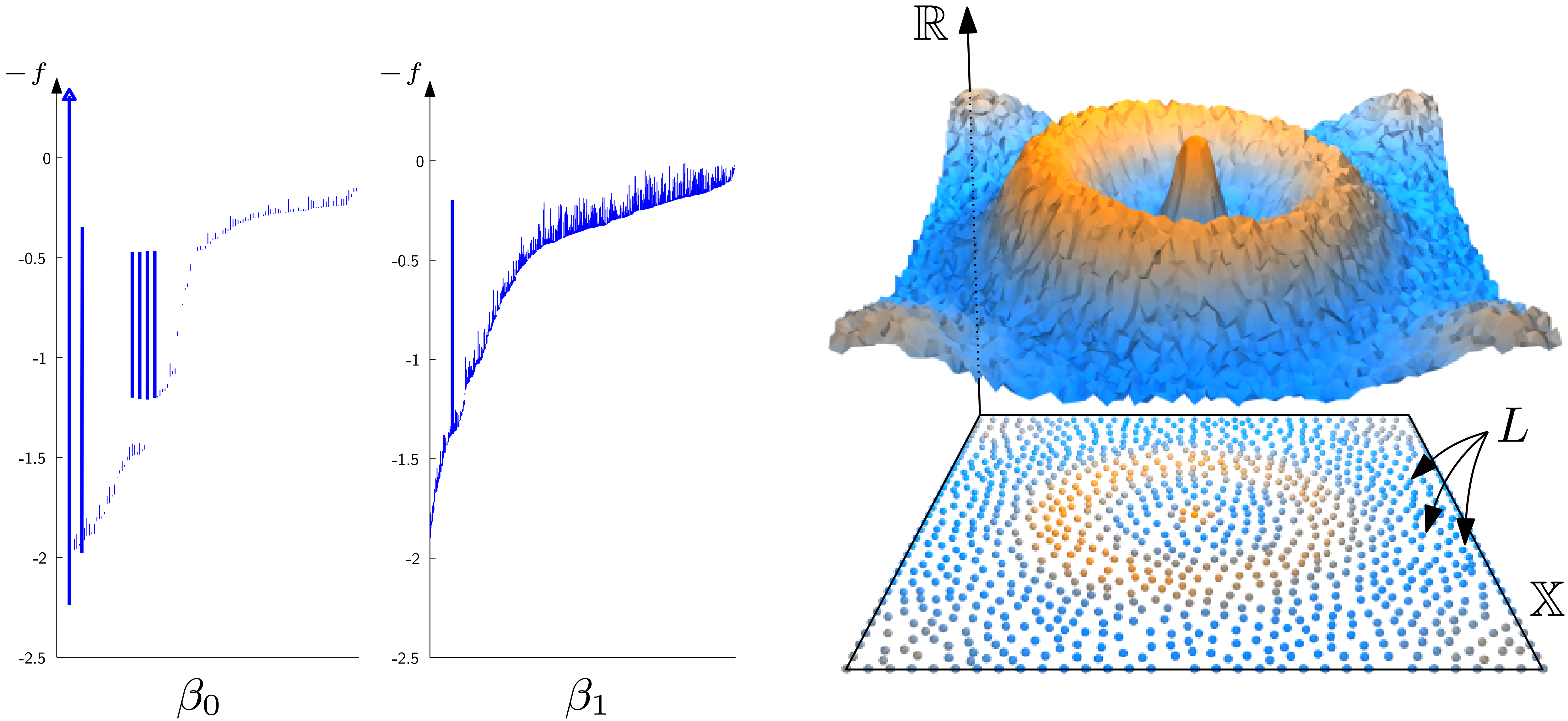
\end{tikzcd}$$

Pairs of Rips Complexes

Assumptions: $\mathbb{X}$ Riemannian manifold, $f : \mathbb{X} \to \mathbb{R}$ c-Lipschitz, L geodesic ε-cover of $\mathbb{X}$, for some unknown $\varepsilon > 0$.



(6 prominent peaks) (ring-shaped basin of attraction)

Basins of Attraction

Goal: approximate basins of attraction of significant peaks of f

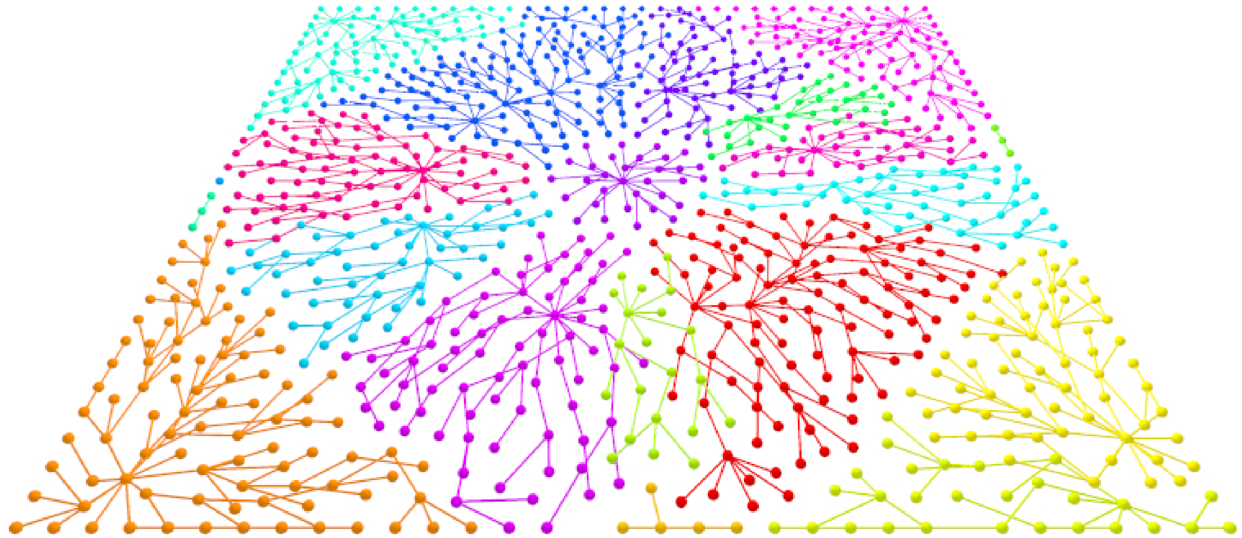
$\Rightarrow$ segmentation/clustering of point cloud L

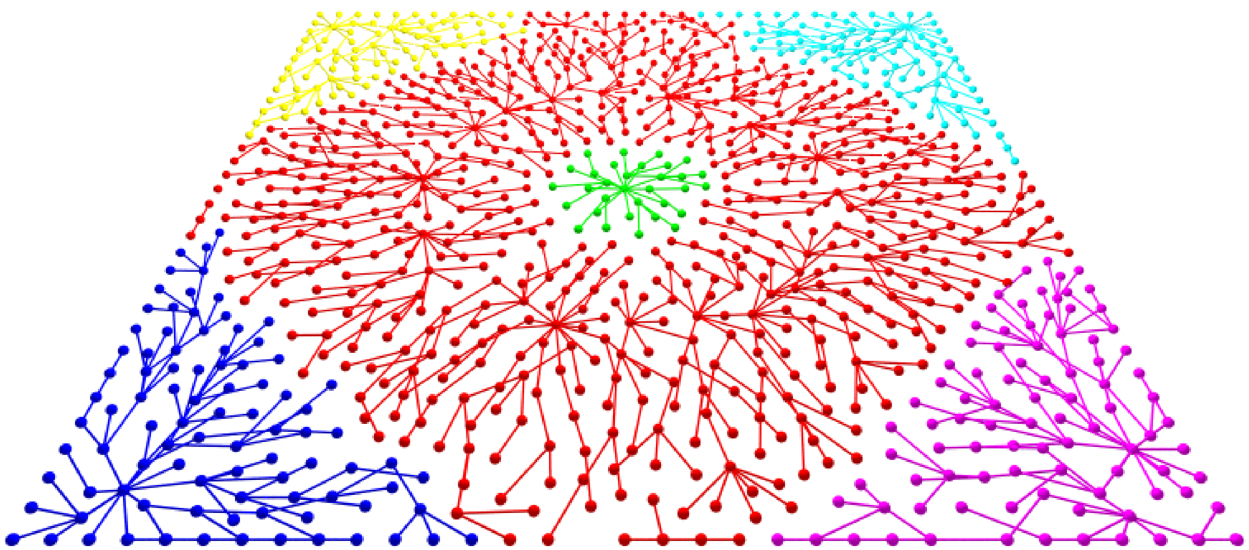
Approach:

- rough approximation of gradient of f within Rips graph,
- merge clusters according to 0-dimensional barcode.

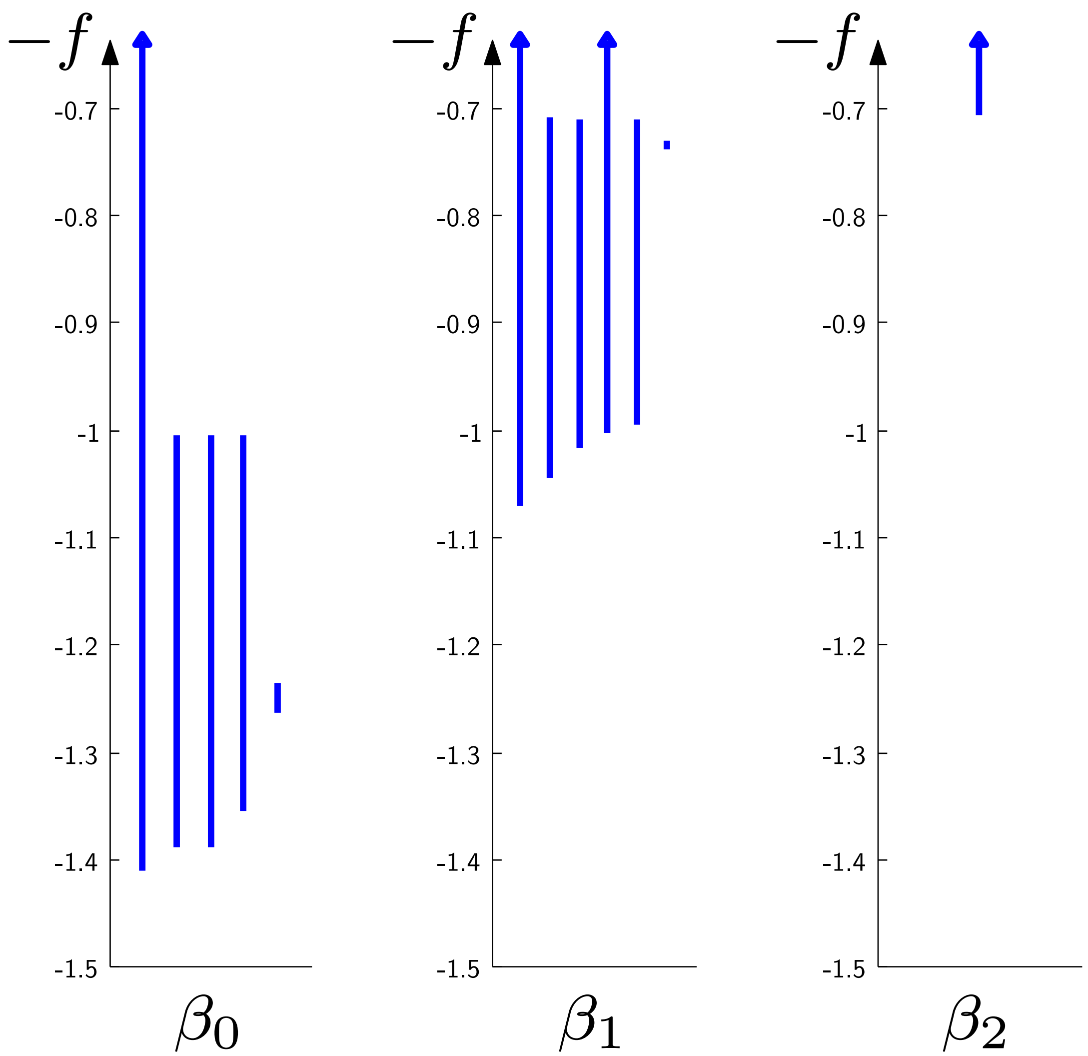
$\rightarrow$ union-find data structure

(see SODA '09 paper for details)

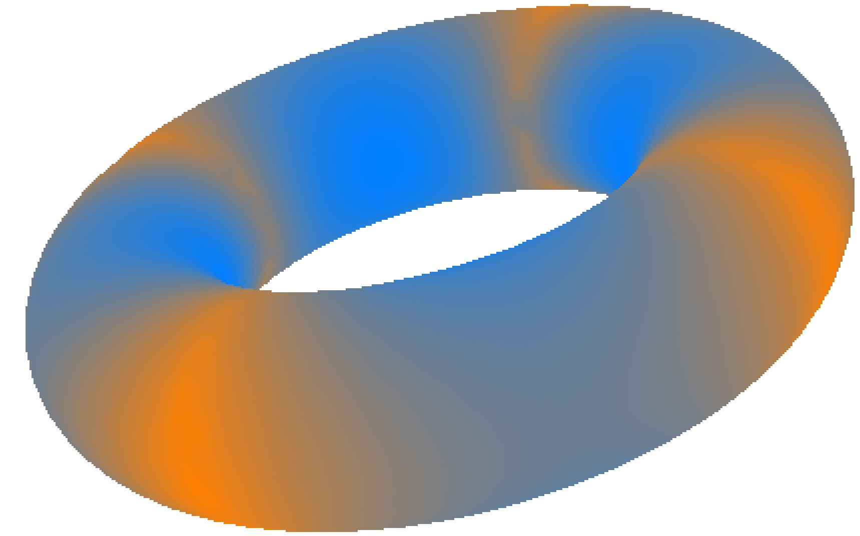




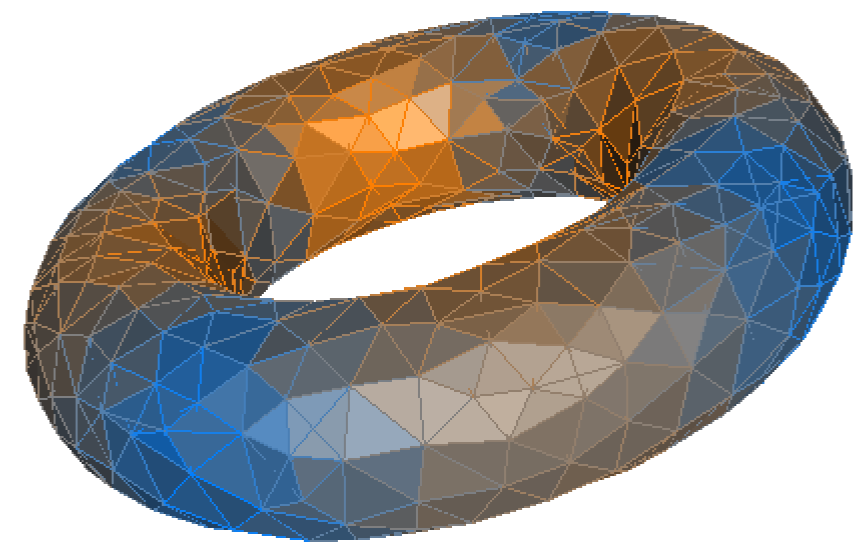
Some Results



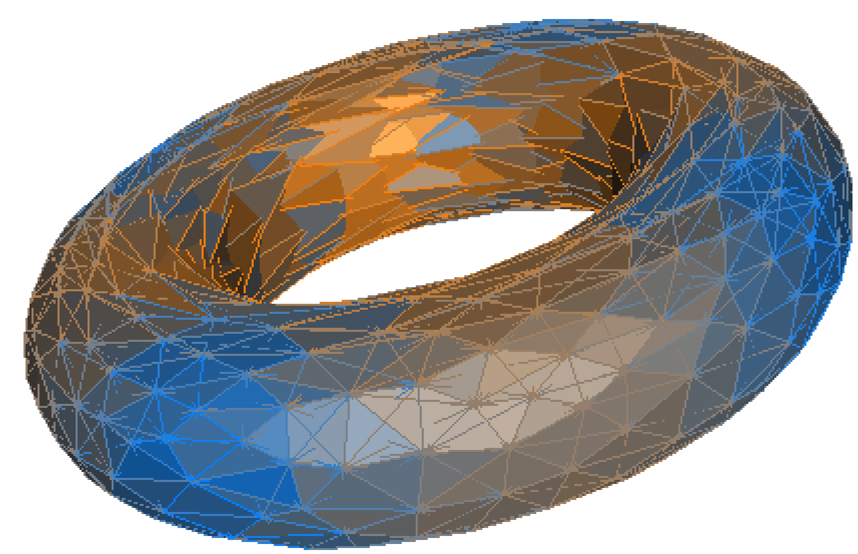
$(\mathbb{X}, f)$



$\mathcal{R}^{\delta}(L)$



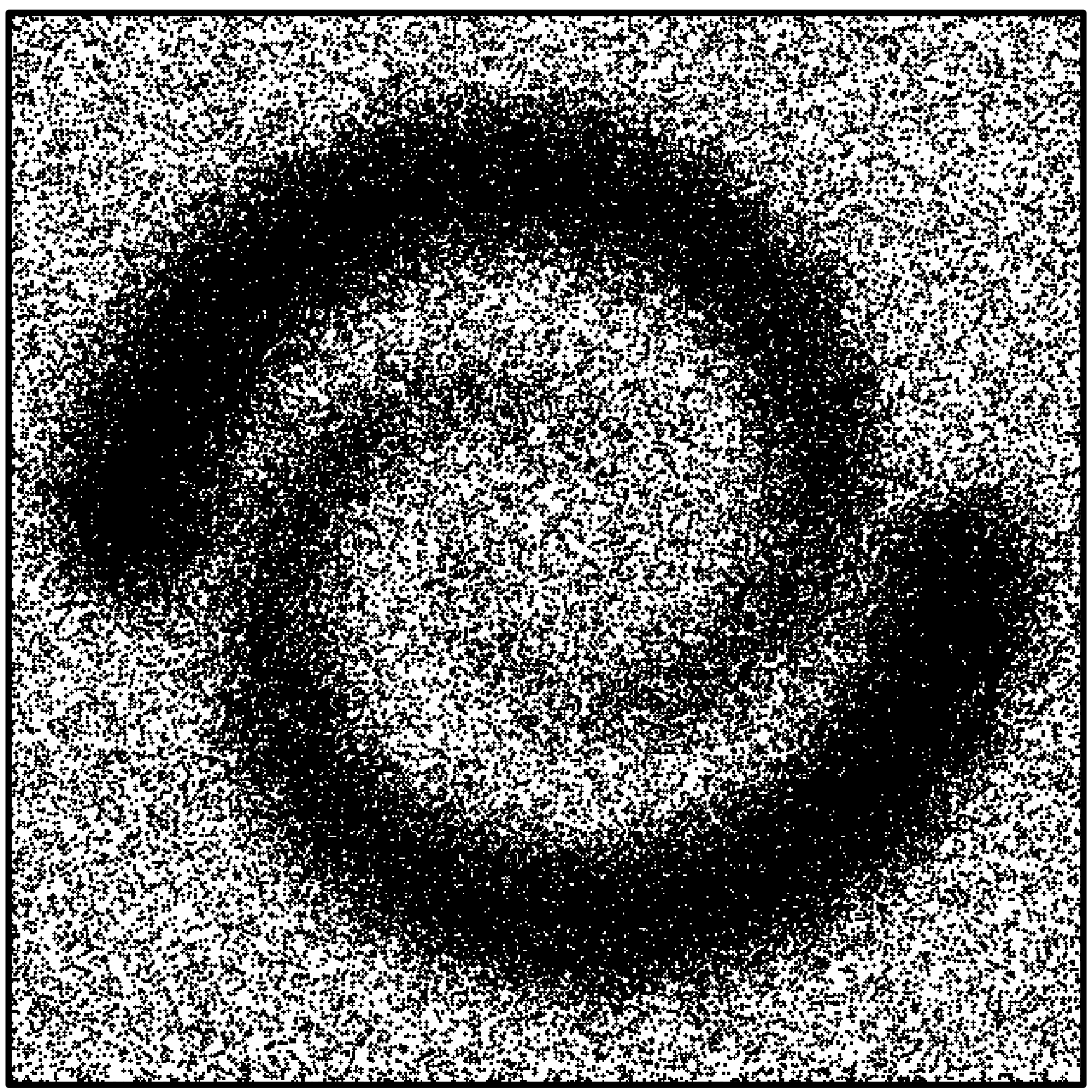
$\mathcal{R}^{2\delta}(L)$

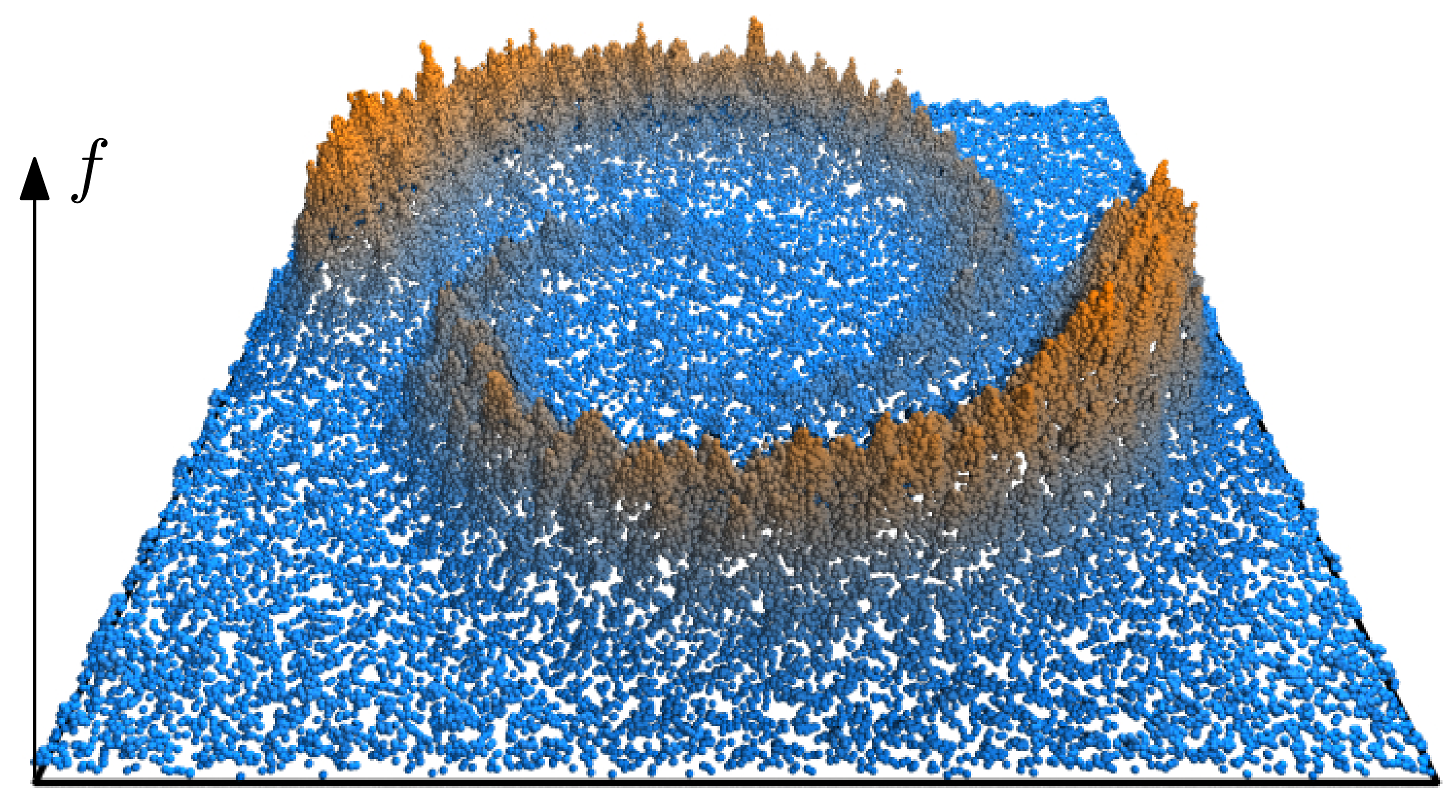


Some Results

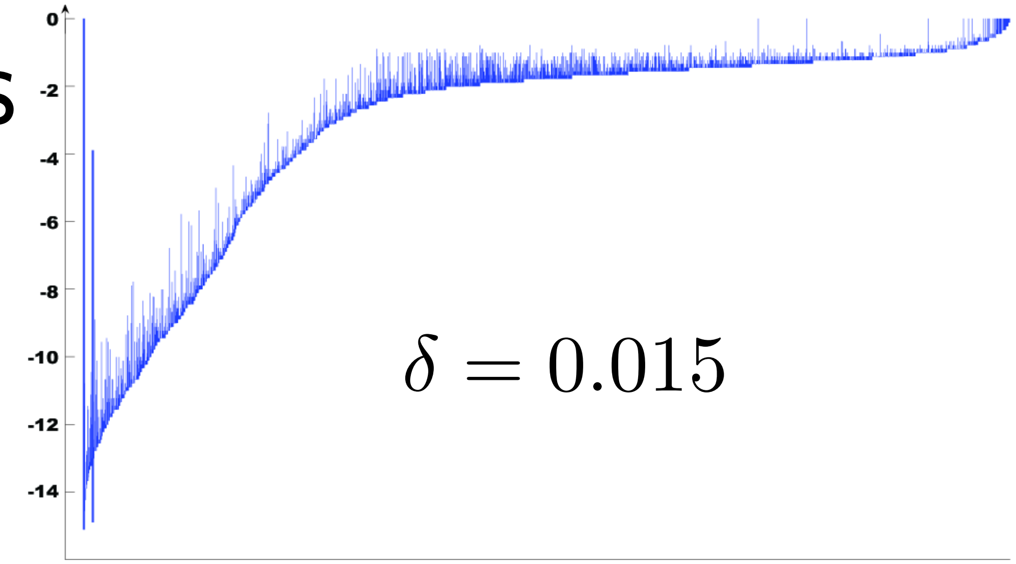
Input: $\mathbb{X} = [0,1]^2$; $|L| = 100,000$;

$f = \#$ { data pts in fixed-radius ball }





Some Results



Input: $\mathbb{X} = [0,1]^2$; $|L| = 100,000$;

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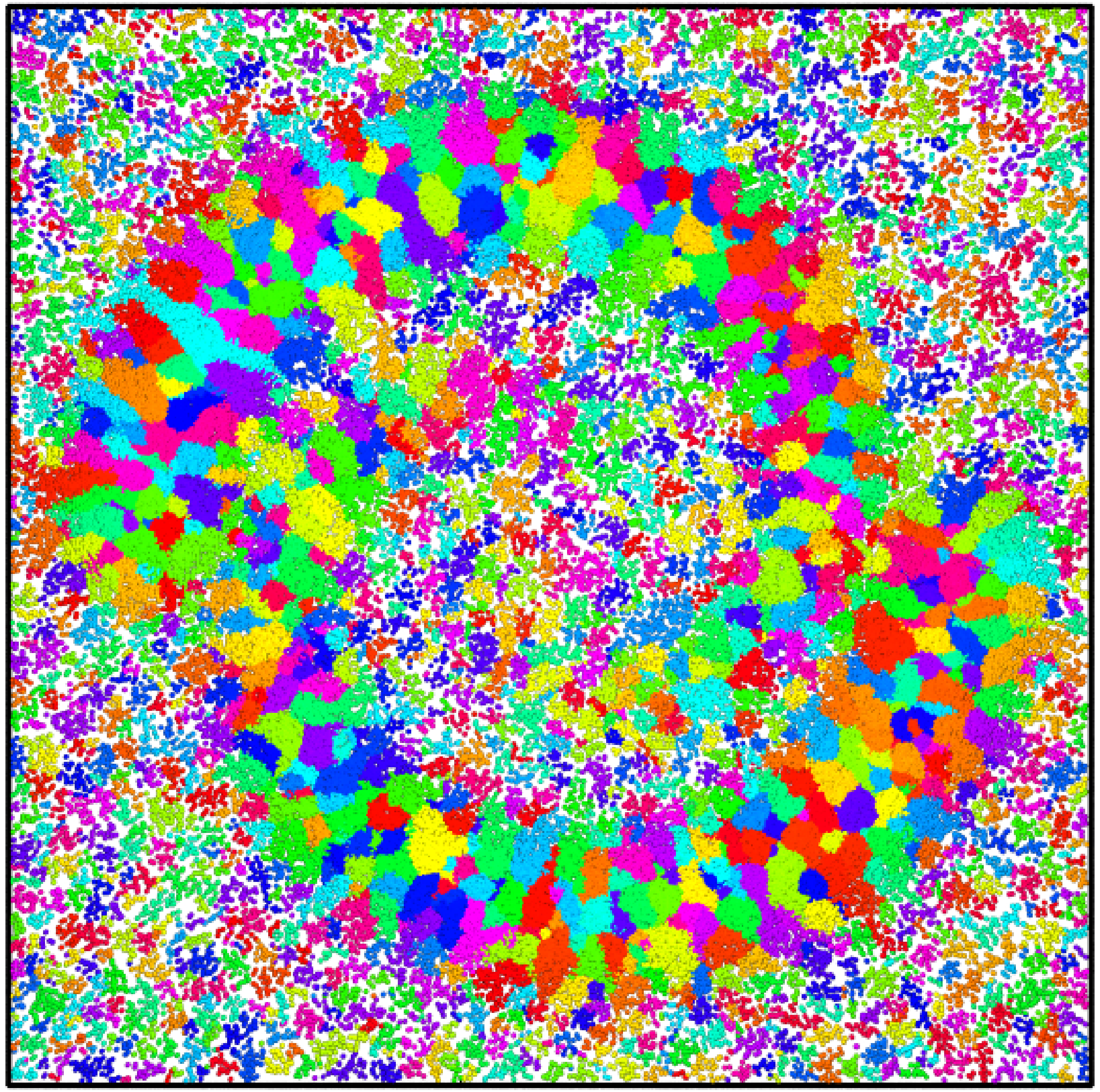
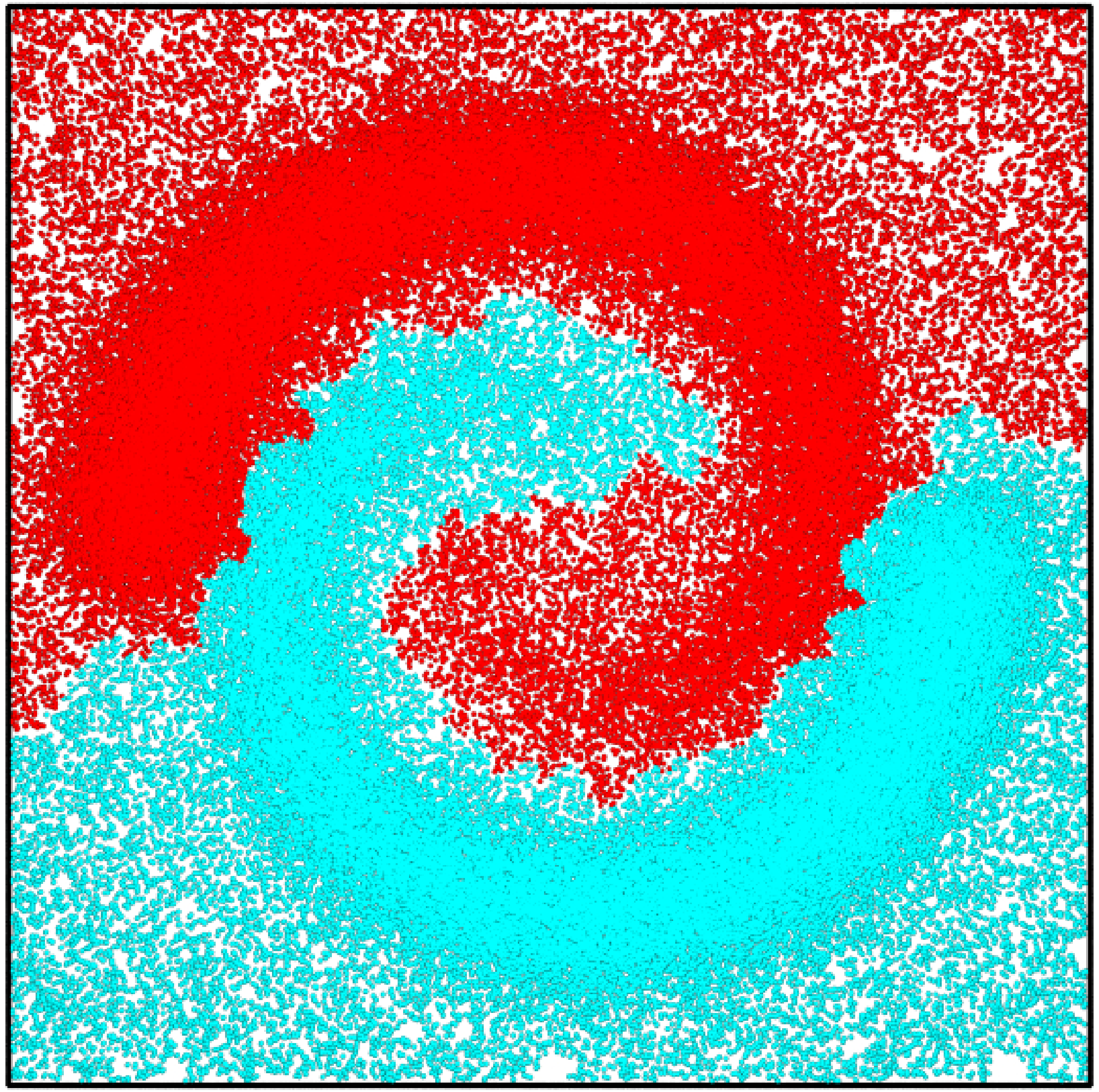
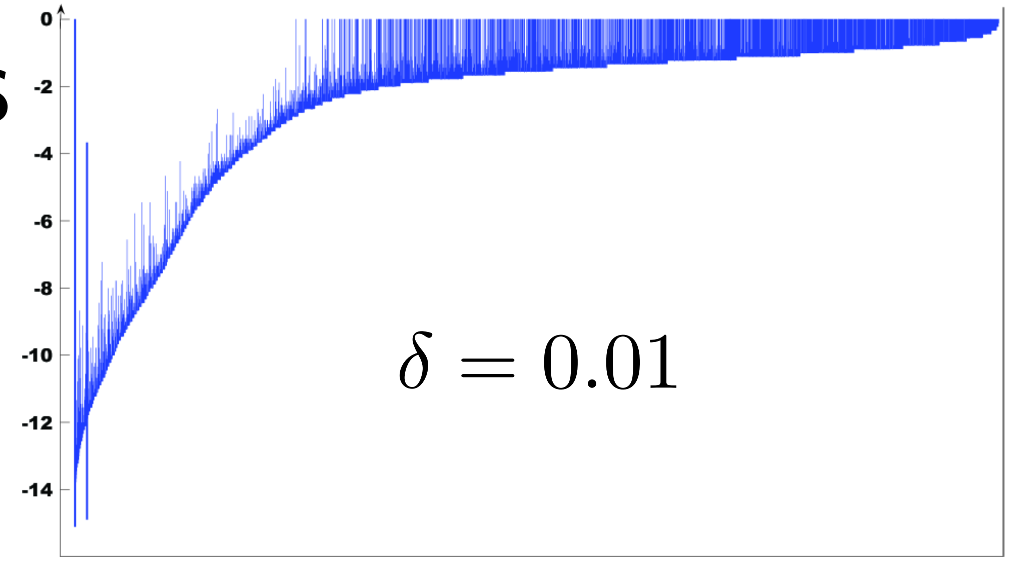




Some Results



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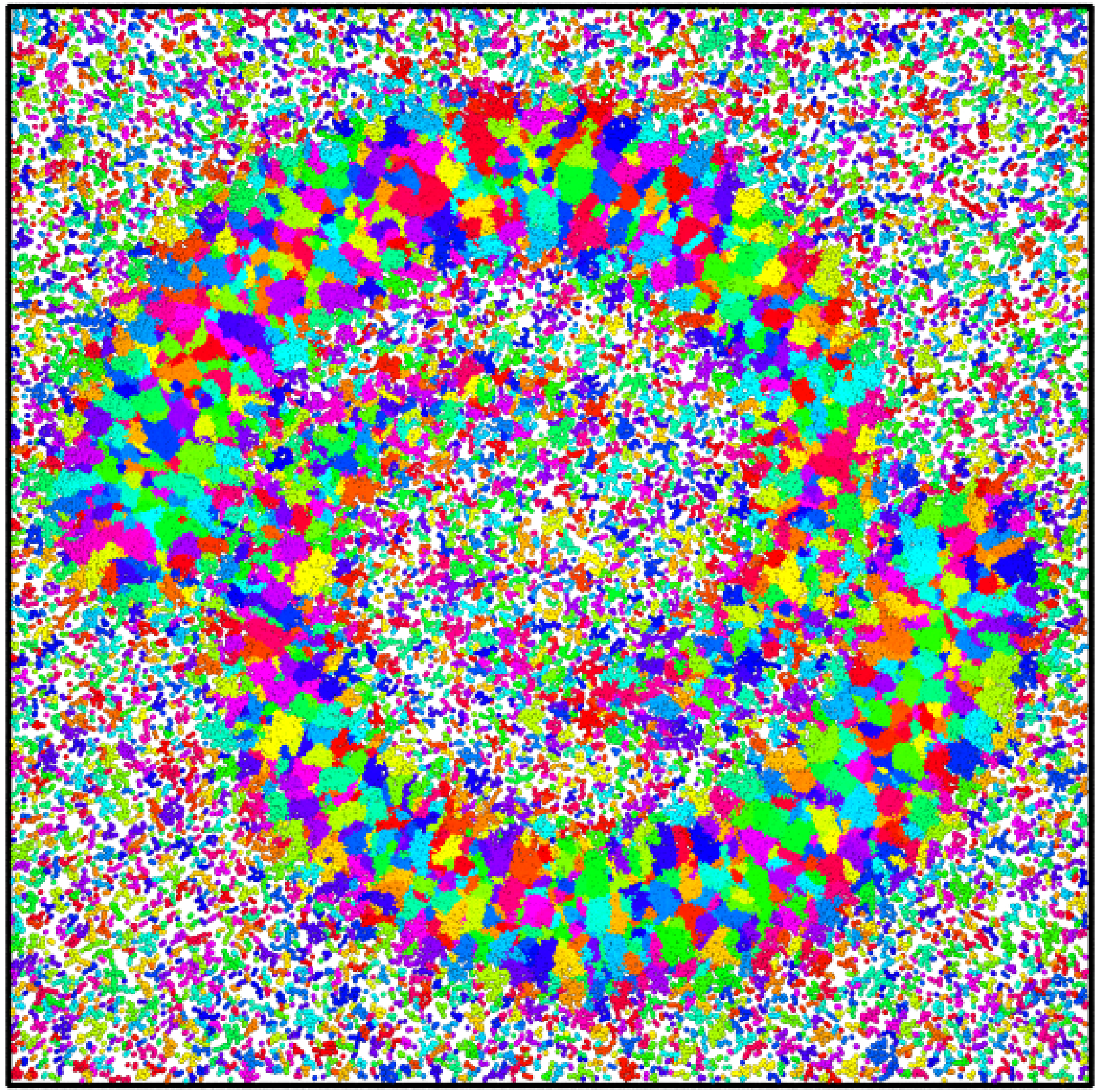

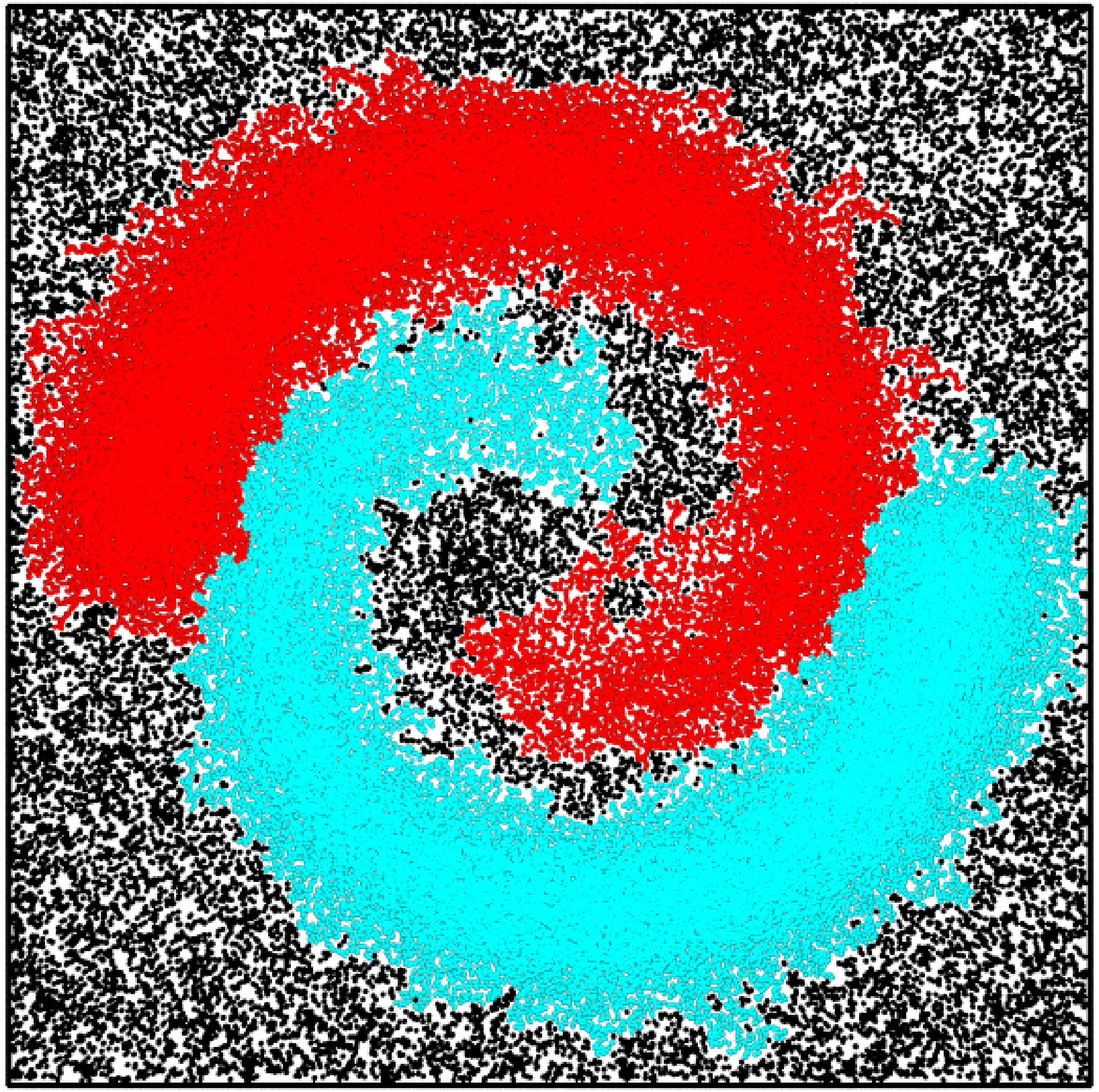

Take-Home Message(s)

- Extension of the persistence paradigm to non-triangulated or non-triangulable spaces.

- New applications of the persistence-based approach: clustering. shape segmentation, ... any problem cast into the one of finding significant peaks in some scalar field over a sampled space.

- Extensions:
  - robustness to noise,
  - partially-sampled spaces,
  - time-varying scalar fields,
  - non-manifold Alexandrov spaces.